TRACY'S NEW SERIES.

AN

ELEMENTARY

ARITHMETIC,

CONTAINING

EXTENSIVE EXERCISES FOR THE SLATE.

BY C. TRACY, A.M.,

PRINCIPAL OF CLASSICAL INSTITUTE, NEW YORK.

PHILADELPHIA:

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INTRODUCTION.

In the preparation of the following treatise, a special object in view has been, to present a work containing *primary exercises* sufficient for the scholar in his first efforts upon the slate.

There is no good reason why the learner should not acquire the same facility, promptness, and accuracy, in combining figures, that he does in *reading his native language;* the result in each case being attained by similar instrumentalities.

If however we compare the means employed for securing each result, we shall not be surprised that what is to so great an extent secured in the one case, often proves a failure in the other. The child's first object, in his educational course, is to learn to read, and his efforts for the attainment of this result are not discontinued during his entire course of study. But how is it with regard to arithmetic? Take for illustration the exercise of *adding*, a department of arithmetical operations probably more used in business, than all other numerical operations together. In a matter of such importance, the child should acquire the utmost facility and accuracy. Why, then, is the attainment so seldom made? We apprehend the difficulty lies in the want of equally extensive exercises in combining numbers. Most arithmetics are very limited in practical work in this important department of numerical operations. To remedy this difficulty, was in part the design with which this primary treatise was commenced.

In enumerating some of the more prominent features introduced, we shall restrict our remarks to the more elementary *portions* of the work.

The system of *Numeration* introduced will be found somewhat peculiar. In the first place, numbers are resolved into the natural periods of units, tens, hundreds, thousands, &c.; then, by combining periods of units and tens, all numbers between 1 and 99, inclusive, are formed; also, by combining periods of units, tens, and hundreds, all numbers between 1 and 999, inclusive, are formed; and so on through the higher denominations. The pupil is thus instructed not merely in the art of reading figures, but of forming and analyzing numbers.

In *Simple Addition*, a large amount of practical work is presented for the scholar in his first efforts upon the slate. A peculiarity in this part of the system consists in adding by *combination;* by which mental labor is much diminished, while increased expedition and accuracy are secured. This is a matter of practical importance, both in the schoolroom, and the business operations of every-day life.

A second peculiarity will be found in the application of the principle of *Subtraction*. After the introductory mental and slate exercises, a *business application* of the principle is introduced by natural and obvious gradations. The first consists in subtracting *a single number* from the amount of *several numbers;* the second, in subtracting the *amount of several numbers from a single number;* and the third, in subtracting the *sum of several numbers from the amount of several others*. These three distinct subdivisions, embrace every possible application of the principle of subtraction, and for each, specific directions, illustrations, and explanations are given.

The same general system is carried out in the application of the other simple rules.

The simple rules are succeeded by the *decimal system*, which is regarded as an extension of the system of integral numbers, to numbers of less than integral value, and susceptible of being used in immediate connection with them. This section is followed by *Federal numbers*, and this, in turn, by *Fractional numbers*, of each of which a practical application is made. The fundamental principles of numbers are thus introduced and applied to practical purposes in every possible form.

This treatise, although designed especially for the pupil in his first efforts on the slate, is believed to embrace an amount of arithmetical matter, sufficient for the practical purposes of life. It is also *strictly analytical*.

In conclusion, we add a single remark. Judging from the character of many books designed for schools, especially of such as are of a more primary character, the conclusion is natural, that their authors regard it as an object of the first importance, to adapt the language employed, *strictly to the present capacities of the child;* that is, to adopt the child's vocabulary as the language of their books. In this particular, we beg leave most respectfully to differ. The child is not always to remain such. The law of his nature is *progress*,—a gradual development of intellectual energy. Hence he needs something above his present capacity; something that shall elicit his intellectual energies. His course in life is not to be a railroad level;—nor is intellectual speed the chief object to be attained. There must be substantial growth, and especially in the acquisition and use of language. In the character of the language adopted in the preparation of the present treatise, we have had in view this acquisition; and, while we have written for the child, we have preferred to employ language adapted to the gradual development of youthful powers; and should the scholar find it necessary to consult his teacher or his dictionary occasionally, to learn the true import of a word, we apprehend the time so employed will not be valueless in its result.

C. TRACY.

New York, 1850.

CONTENTS.

ARITHMETIC.

§ 1. Arithmetic teaches the nature and use of figures.

1. Figures are characters employed to represent numbers.

2. Operations by numbers, when performed in the mind unaided by the pen or pencil, constitute *mental arithmetic.*

3. When, however, these operations are expressed by written characters, the work is termed *written arithmetic.*

4. Both mental and written arithmetic embrace the same operations; viz. *Numeration, Addition, Subtraction, Multiplication, and Division.*

NUMERATION.

§ 2. Numeration teaches the art of reading figures.

SERIES FIRST, *composed of units.*

1. A unit consists of a single object, not limited in its kind.

2. Hence, unit figures are the representatives of in-

1. What does arithmetic teach? What are figures? What, mental arithmetic? What, written arithmetic? What do mental and written arithmetic both embrace?

2. What does numeration teach? A unit consists of what? Unit figures represent what?

dividual objects. They are presented in the following table:—

TABLE OF UNITS.

1. One unit is counted, *one;* and written - - - - - - - - 1.
2. Two units are counted, *one, two;* and written - - - - - 2.
3. Three units are counted, *one, two, three;* and written - - 3.
4. Four units are counted, *one, two, three, four;* and written - 4.
5. Five units are counted, *one, two, three, four, five;* and written - - - - - - - - - - - - - - - - - - 5.
6. Six units are counted, *one, two, three, four, five, six;* and written - - - - - - - - - - - - - - - - - 6.
7. Seven units are counted, *one, two, three, four, five, six, seven;* and written - - - - - - - - - - - - - 7.
8. Eight units are counted, *one, two, three, four, five, six, seven, eight;* and written - - - - - - - - - - - 8.
9. Nine units are counted, *one, two, three, four, five, six, seven, eight, nine;* and written - - - - - - - - - - 9.

3. The above is called the table of *units*, because each figure *represents single objects.*

4. The single objects represented by each figure are determined by *counting that number.*

§ 3. How many whole objects are represented by figure 1? by figure 2? figure 3? figure 4? figure 5? figure 6? figure 7? figure 8? figure 9? What character is employed besides the nine given in the above table? *Ans.* The cipher. What is the form of the cipher? It resembles the letter O. Has it any value when standing alone? *Ans.* None. How can you determine the number of whole objects expressed by each figure? *Ans.* By counting the number represented by that figure.

Write on the slate or board 3, 5, 7, 9, 2, 4, 6, 8, 1.

One, how written?—counted? Two, how written?—counted? Three, how written?—counted? Four, how written?—counted? Five, how written?—counted? Six, how written?—counted? Seven, how written?—counted? Eight, how written?—counted? Nine, how written?—counted? Why called table of units? How determine the objects represented by each figure?

SERIES SECOND, *composed of tens.*

§ 4.—1. One ten equals ten units.

2. The figures that express units are made to represent tens *by placing a cipher on the right hand of each.*

TABLE OF TENS.

One ten is written	10	Six tens, or sixty,	60
Two tens, or twenty,	20	Seven tens, or seventy,	70
Three tens, or thirty,	30	Eight tens, or eighty,	80
Four tens, or forty,	40	Nine tens, or ninety,	90
Five tens, or fifty,	50		

§ 5.—1. Each number in the above table expresses *tens;* that is, each of them represents *ten times* as many single objects as when written without a cipher, as in the table of units.

2. It must also be remembered that the *cipher* has given each figure this increased value. Hence, the cipher, although it has no value when standing alone, increases the value of the other figures in a tenfold ratio,—that is, makes them ten times as large, when placed on their right.

3. Hence, also, any figure, having a cipher on its right, is said to occupy *the place of tens*, instead of the place of units, as when standing alone.

4. This increased value is called the *local value* of a figure.

§ 6. How many single objects or units, and how many tens are represented by 10? *Ans.* Ten objects, but only one ten. How many objects and how many tens are repre-

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4. What does 1 ten equal? What figures represent tens? How made to represent tens? One ten, how written? Two tens, how written? Three tens, how written? Four tens, how written? Five tens, how written? Six tens? Seven tens? Eight tens? Nine tens?

5. What does each number express? How many times more objects than in table of units? What has increased the value? In what ratio does the cipher increase the value of figures? What is meant by tenfold ratio? What place is a figure said to occupy when one cipher stands on its right? What is the increased value a figure receives, as it changes its place, called?

sented by 20? *Ans.* Twenty objects and two tens. By 30? by 40? by 50? by 60? by 70? by 80? by 90?

Write on the slate 1 ten, (10,) 2 tens, 7 tens, 9 tens, 2 tens, 3 tens, 5 tens, 4 tens, 6 tens, 8 tens.

§ 7.—1. In the table of units we learn to express *units* only, and in the table of tens to express *tens* only.

2. We shall next investigate the manner of expressing numbers between ten and twenty, twenty and thirty, thirty and forty, forty and fifty, &c.

3. Figures when standing alone, that is, when there is no figure on their right, express *units* only, (§ 2;) when, however, they are made to occupy a second place towards the left, by placing ciphers on their right, they invariably represent *tens*.

4. Hence the *first*, or *right-hand place*, is called *units'* place, and the *second place*, or place next to the left, is called *tens'* place.

5. Obviously, therefore, to express any number composed of both units and tens, we unite in one expression the given units and tens, placing the units always *on the right of the tens*, that is, in the place occupied by the cipher in the table of tens.

Application of § 7. What number is produced by uniting 1 ten and 1 unit? *Ans.* 11. 1 ten and 2 units? *Ans.* 12. 1 ten and 3 units, &c. What number is produced by uniting 2 tens and 5 units? *Ans.* 25. 2 tens and 7 units? 2 tens and 8 units? 2 tens and 9 units? 3 tens and 3 units? 3 tens and 4 units? 3 tens and 7 units? 3 tens and 9 units? 4 tens and 7 units? 4 tens and 8 units? 4 tens and 1 unit? 4 tens and 5 units? 4 tens and 3 units? 4 tens and 9 units? 5 tens and 7 units? 5 tens and 6 units? 5 tens and 9 units? 5 tens and 7 units? 6 tens and 8 units? 6 tens and 5 units? 6 tens and 3 units? 7 tens and 4 units? 7 tens and 7 units? 7 tens

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7. What do we learn to express in table of units? What, in table of tens? Figures standing alone express what? What when a cipher stands on their right? What is the first place on the right called? What is the second place on the left called? How do we express a number composed of both units and tens?

and 5 units? 8 tens and 6 units? 8 tens and 9 units? 8 tens and 3 units? 9 tens and 9 units? 9 tens and 7 units? 9 tens and 3 units? 9 tens and 4 units? 9 tens and 1 unit? 9 tens and 0 units?

Combine in one expression (use the slate) 3 tens, 7 units; 4 tens, 2 units; 1 ten, 9 units; 5 tens, 7 units; 6 units, 3 tens; 8 units, 7 tens; 3 units, 9 tens; 1 unit, 8 tens; 7 units, 3 tens; 1 unit, 6 tens.

SERIES THIRD, *composed of hundreds.*

§ 8. One hundred equals ten tens, or a hundred units. Two hundred equals twenty tens, or two hundred units. Three hundred equals thirty tens, or three hundred units, &c.

TABLE OF HUNDREDS.

One hundred	is written	100	Six hundred	is written	600
Two hundred	"	200	Seven hundred	"	700
Three hundred	"	300	Eight hundred	"	800
Four hundred	"	400	Nine hundred	"	900
Five hundred	"	500			

§ 9.—1. Each number in the above table represents *ten times* as many objects as the corresponding numbers in the table of tens, and a hundred times as many as in the table of units. This increase in value is produced by annexing another cipher.

2. The figures 1, 2, 3, 4, 5, &c., standing alone, express units or single objects; with one cipher on the right, they express tens; and with two ciphers, they express hundreds.

3. Hence, any figure, when it has two ciphers on its right, is said to occupy the place of hundreds.

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8. One hundred equals how many tens?—how many units? Two hundred, how many? Three hundred, how many?

9. What does each number of series 3d represent? How is this increase produced? What do the figures 1, 2, 3, 4, &c., standing alone, express? What, with one cipher on the right? With two ciphers? What place does a figure, having two ciphers on the right, occupy?

§ 10. How many hundreds, how many tens, and how many units are represented by 100, (that is, by 1 and 00?) *Ans.* 1 hundred, 10 tens, or 100 units. How many of each by 200? How many of each by 300? by 400? by 500? by 600? &c.

§ 11.—1. Our next object will be to combine units, tens, and hundreds in *one expression*. By this combination *we shall be able to express any number from one to nine hundred and ninety-nine, inclusive.*

2. Each figure standing alone expresses *units*, (§ 2,) and with a cipher on its right, it expresses tens.

3. When, also, one significant figure is placed on the right of another, they together express a *combination of units and tens.* (§ 7.)

4. Each figure also expresses *hundreds*, when two ciphers are placed on its right. Hence, the first, or right-hand place, is the place of units; then, advancing to the left, the *second* is the place of *tens*, and the *third* the place of hundreds.

5. To combine units, tens, and hundreds in one expression, *we have then only to give each figure its appropriate place;* viz. to place the *ten's figure on the left of the unit figure, and the hundred's figure on the left of the tens.*

§ 12. What number is produced by combining in one expression 1 hundred, 7 tens, and 6 units? 2 hundreds, 8 tens, and 5 units? 4 hundreds, 9 tens, and 7 units? 7 hundreds, 4 tens, and 9 units? 3 hundreds, 5 tens, and 4 units? 2 hundreds, 6 tens, and 4 units? 5 hundreds, 7 tens, and 9 units? 6 hundreds, 4 tens, and 8 units? 1 hundred, 9 tens, and 6 units? 4 units, 3 tens, and 7 hundreds? 3 tens, 7 hundreds, and 5 units? 6 units, 2 hundreds, and 5 tens? 4 tens, 9 hundreds, and 7 units? 4 tens,

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11. What numbers can be expressed by a combination of units, tens, and hundreds? When does a figure express units? When tens? When do figures express units and tens combined? When does each figure express hundreds? What is the right-hand place? the second place? the third? How do we combine units, tens, and hundreds in one expression?

8 units, and 9 hundred? 8 tens, 9 units, and 5 hundred? 6 units, 5 hundred, and 7 tens?

SERIES FOURTH, *composed of thousands.*

§ **13.** One thousand equals ten hundred, a hundred tens, or a thousand units. Two thousand equals twenty hundred, two hundred tens, or two thousand units. Three thousand equals thirty hundred, three hundred tens, or three thousand units, &c.

TABLE OF THOUSANDS.

One thousand is written		1000	Six thousand is written		6000
Two thousand	" -	2000	Seven thousand	" -	7000
Three thousand	" -	3000	Eight thousand	" -	8000
Four thousand	" -	4000	Nine thousand	" -	9000
Five thousand	" -	5000			

§ **14.**—1. The numbers presented in the former tables are here again increased tenfold, and this increase is effected in the same manner as before; viz., by placing a cipher on the right of each of those numbers. Each cipher placed on the right of any number removes each figure of that number one place further to the left, and by so removing it, increases its value in a tenfold ratio,—that is, makes its value ten times as great as before.

2. We hence perceive, that each and every figure has a *local* value, that is, a value depending upon the place it occupies, besides its own *specific* value, as shown in Table of Units, (§ 2;) also that by employing only ten characters, viz., 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, any number, however large, can be expressed.

§ **15.** What number is produced by combining in one expression 1 thousand, 8 hundred, 9 tens, and 3 units?

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13. One thousand equals how many hundred? tens? units? Two thousand equals how many hundred? tens? units? Three thousand equals how many hundred? tens? units? One thousand how written? two, how written? three, how written? &c.

14. How are previous numbers here increased? how is this increase effected? What is the effect of each cipher? how is the value affected? What value has each figure? depending on what? what other value? How many characters required to express all numbers?

5 thousand, 3 hundred, 9 tens, and 7 units? 4 thousand, 5 hundred, 9 tens, and 7 units? 9 thousand, 8 hundred, 1 ten, and 8 units? 6 hundred, 5 tens, 8 thousand, and 2 units? 1 unit, 8 hundred, 7 tens, and 3 thousand? 9 tens, 8 thousand, 5 units, and 6 hundred? 5 tens, 2 thousand, 5 hundred, and 6 units? 3 thousand, 3 units, 5 tens, and 8 hundred? 8 tens, 6 units, 9 thousand, and 7 hundred? 7 hundred, 6 units, 5 thousand, and 8 tens? 4 hundred, 6 units, 8 tens, and 5 thousand? 1 ten, 7 units, 1 hundred, and 7 thousand? 5 tens, 6 hundred, 4 thousand, and 7 units? 9 thousand, 3 units, 5 tens, and 8 hundred? 7 hundred, 6 units, 5 tens, and 8 thousand?

Similar questions should be continued till the scholar is master of the subject.

§ **16.**—1. Hundreds and units, thousands and units, thousands and tens, &c., may be combined in one number, *by writing ciphers in the place of the intermediate denominations*. That is, to combine thousands and units in one expression, we write ciphers in the place of tens and hundreds; to combine hundreds and units we write a cipher in the place of tens, &c. Thus three hundred and three is expressed in figures by placing a cipher in the place of tens, as follows: 303; three thousand and three, by placing two ciphers in the place of hundreds and tens, as in the following expression: 3003; and three thousand and thirty by writing ciphers in the place of hundreds and units, thus: 3030, &c.

2. Combine in one number 5 thousand and 7 units; 6 hundred and 4 units; 7 hundred and 4 tens; 3 hundred and 7 tens; 3 thousand and 5 tens; 5 units and 6 thousand; 8 tens and 7 thousand; 5 units, 4 tens, and 7 thousand; 2 tens, 7 units, and 6 thousand; 7 units and 7 hundred; 8 tens, 8 hundred, and 3 thousand; 4 thousand, 6 units, and 5 tens; 3 units, 7 tens, 8 hundred, and 1 thousand; 8 hundred and 5 thousand; 9 tens and 3

16. How may hundreds and units, thousands and units, &c., be combined in one number? To combine thousands and units, where do we write ciphers? where to combine hundreds and units?

thousand; 5 units and 9 hundred; 9 units and 9 thousand.

§ 17.—1. Were we to proceed, we should next form a fifth series by the introduction of a fourth cipher, removing the significant figures 1, 2, 3, 4, &c., one place still further to the left, and thereby again increasing the value of each in a tenfold ratio, by which they become 1 ten thousand, (10000,) 2 ten or 20 thousand, (20000,) 3 ten or 30 thousand, (30000,) 4 ten or 40 thousand, (40000,) &c.

2. Then again by a similar addition,—that is, by the introduction of five ciphers, these same figures become hundreds of thousands;—viz., 1 hundred thousand, (100000,) 2 hundred thousand, (200000,) 3 hundred thousand, (300000,) 4 hundred thousand, (400000,) &c. Advancing a series higher still, and millions are obtained,—expressed by writing six ciphers on the right of the figures, 1, 2, 3, 4, &c., viz., 1 million, (1000000,) 2 million, (2000000,) 3 million, (3000000,) &c.

§ 18.—1. We learn from the previous remarks, that the first or right-hand figure of any number of figures, is *units;* the second from the right, *tens;* the third, *hundreds;* the fourth, *thousands;* the fifth, *tens of thousands;* the sixth, *hundreds of thousands;* the seventh, *millions*, &c. Also that the *local* value of any figure depends upon which of these places it occupies. The simple tens of thousands are written 10000, 20000, 30000, &c; the simple hundreds of thousands, 100000, 200000, 300000, &c; the simple millions, 1000000, 2000000, 3000000, &c., and units, tens, hundreds, thousands, &c., are combined with these, by placing each figure in the place bearing its own name;—that is, units in units' place, tens in tens' place, hundreds in

17. How many ciphers required to form another series? How would the figures 1, 2, 3, &c., be affected? In what ratio would their value be increased? What do they become? What do the significant figures become by the introduction of five ciphers? What is obtained by a series still higher?

18. What is the first or right-hand figure? the second? the third? the fourth? the fifth? the sixth? the seventh? Upon what does the local value of the figure depend? How are simple tens of thousands written? Hundreds of thousands? Millions?

hundreds' place, &c. The following table shows the relative place of each denomination:

Millions.	Hund. of thou.	Tens of thou.	Thousands	Hundreds.	Tens.	Units.
3	7	8	9	6	7	4

2. This number is analyzed as follows:—4 units, 7 tens, 6 hundred, 9 thousand, 8 tens of thousands, 7 hundreds of thousands, and 3 million, and read in the reverse order, viz., 3 million, 7 hundred and 89 thousand 6 hundred and 74.

3. The same order of increase continues as we advance from the right to the left, through the denominations higher than millions. Advancing from millions, these denominations are millions, tens of millions, hundreds of millions; billions, tens of billions, hundreds of billions; trillions, tens of trillions, hundreds of trillions; quadrillions, tens of quadrillions, hundreds of quadrillions; quintillions, tens of quintillions hundreds of quintillions, &c.

COMPLETE NUMERATION TABLE.

Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
8	6	5,	6	2	4,	7	4	6,	3	4	6,	9	8	7,	4	5	6.

4. This table by no means contains all possible denominations. They may be continued to almost any extent; but the above embraces sufficient for all practical purposes. Indeed it is seldom that we employ more than nine of the figures in the above table.

How are the several denominations combined? How is the given number analyzed? How read? What is the order of increase for the higher denominations? What are the denominations? Are all denominations embraced in the table? How far may they be continued?

§ **19.** A practical exercise will conclude numeration. What number is produced by combining in one expression 6 tens, 5 units, 7 hundred, and 8 thousand? by combining 5 thousand, 3 units, 5 tens, 6 hundred, and 2 tens of thousands? 4 tens, 8 tens of thousands, 6 units, and 5 hundred? 5 hundred, 9 thousand, and 9 units? 3 units, 5 hundred, 7 thousand, and 9 tens of thousands? 2 hundred, 2 hundreds of thousands, and 5 units? 5 tens, 8 tens of thousands, and 9 hundred? 6 thousand, 5 tens, 6 units, and 5 million? 5 tens of millions, and 8 tens of thousands? 5 units, 6 tens, 7 hundred, 8 thousand, 9 tens of thousands, and 1 million? 3 million, 3 units, and 8 tens? 5 hundreds of thousands, 3 units, and 7 tens? 4 hundred, 6 tens of thousands, 8 tens, and 1 unit? 9 units, 3 hundred, 7 tens of thousands, and 9 million? 8 thousand, 5 units, 6 hundreds of thousands, and 7 tens? 9 tens, 8 hundred, 7 tens of thousands, 9 tens of millions, and 6 units? 4 hundred, 7 tens, 8 tens of thousands, 5 units, and 7 thousand?

The above and the preceding exercises of a similar character, should be written down by the pupil upon the slate. The more successful way for him to operate will be, first to write down a table similar to the following, and then write down the several denominations in their appropriate places, and fill the intermediate, unoccupied places with ciphers. Let it be required to write down the following denominations and determine the number produced, viz., 7 tens, 8 hundred, 9 tens of thousands, 4 units, and 9 hundreds of thousands. The following is the appropriate work:—

Quintillions.	Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
													9	9	0	8	7	4

19. What is the more successful way of expressing the numbers *given?*

ADDITION.

§ **20.**—1. Addition consists in uniting two or more numbers into one.

2. Of numbers representing particular objects, only those expressing the *same kind* of objects can be added so as to form one number.

§ **21.**—1. The following table of Addition requires to be first made perfectly familiar.

2. The addition of any two or more numbers is indicated by the following character, viz., +; that one number equals another number; or one set of numbers, another set of numbers, is indicated by two parallel lines written thus: =. The character + is read *plus*, (meaning more,) and the character = is read *equals*.

ADDITION TABLE.

1+ 0= 1	2+ 0= 2	3+ 0= 3	4+ 0= 4
1+ 1= 2	2+ 1= 3	3+ 1= 4	4+ 1= 5
1+ 2= 3	2+ 2= 4	3+ 2= 5	4+ 2= 6
1+ 3= 4	2+ 3= 5	3+ 3= 6	4+ 3= 7
1+ 4= 5	2+ 4= 6	3+ 4= 7	4+ 4= 8
1+ 5= 6	2+ 5= 7	3+ 5= 8	4+ 5= 9
1+ 6= 7	2+ 6= 8	3+ 6= 9	4+ 6=10
1+ 7= 8	2+ 7= 9	3+ 7=10	4+ 7=11
1+ 8= 9	2+ 8=10	3+ 8=11	4+ 8=12
1+ 9=10	2+ 9=11	3+ 9=12	4+ 9=13
1+10=11	2+10=12	3+10=13	4+10=14
1+11=12	2+11=13	3+11=14	4+11=15
1+12=13	2+12=14	3+12=15	4+12=16

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20. What is addition? What numbers only can be added? How is the addition of numbers indicated? How is the equality of numbers indicated? How is + read? How is = read?

5+ 0= 5	6+ 0= 6	7+ 0= 7	8+ 0= 8
5+ 1= 6	6+ 1= 7	7+ 1= 8	8+ 1= 9
5+ 2= 7	6+ 2= 8	7+ 2= 9	8+ 2=10
5+ 3= 8	6+ 3= 9	7+ 3=10	8+ 3=11
5+ 4= 9	6+ 4=10	7+ 4=11	8+ 4=12
5+ 5=10	6+ 5=11	7+ 5=12	8+ 5=13
5+ 6=11	6+ 6=12	7+ 6=13	8+ 6=14
5+ 7=12	6+ 7=13	7+ 7=14	8+ 7=15
5+ 8=13	6+ 8=14	7+ 8=15	8+ 8=16
5+ 9=14	6+ 9=15	7+ 9=16	8+ 9=17
5+10=15	6+10=16	7+10=17	8+10=18
5+11=16	6+11=17	7+11=18	8+11=19
5+12=17	6+12=18	7+12=19	8+12=20

9+ 0= 9	10+ 0=10	11+ 0=11	12+ 0=12
9+ 1=10	10+ 1=11	11+ 1=12	12+ 1=13
9+ 2=11	10+ 2=12	11+ 2=13	12+ 2=14
9+ 3=12	10+ 3=13	11+ 3=14	12+ 3=15
9+ 4=13	10+ 4=14	11+ 4=15	12+ 4=16
9+ 5=14	10+ 5=15	11+ 5=16	12+ 5=17
9+ 6=15	10+ 6=16	11+ 6=17	12+ 6=18
9+ 7=16	10+ 7=17	11+ 7=18	12+ 7=19
9+ 8=17	10+ 8=18	11+ 8=19	12+ 8=20
9+ 9=18	10+ 9=19	11+ 9=20	12+ 9=21
9+10=19	10+10=20	11+10=21	12+10=22
9+11=20	10+11=21	11+11=22	12+11=23
9+12=21	10+12=22	11+12=23	12+12=24

MENTAL ARITHMETIC.

§ **22.** Mental Arithmetic consists in performing operations without slate or pencil, the work being done in the mind. The following numbers are to be added mentally before commencing the sums designed for the slate.

NOTE TO TEACHERS. Scholars should be drilled on this and the preceding table till every answer becomes a matter of intuition.

22. In what does mental arithmetic consist?

What is the amount of

1 and 1?	2 and 1?	3 and 1?	4 and 1?
1 and 3?	2 and 3?	3 and 3?	4 and 3?
1 and 2?	2 and 2?	3 and 2?	4 and 2?
1 and 5?	2 and 5?	3 and 5?	4 and 5?
1 and 4?	2 and 4?	3 and 4?	4 and 4?
1 and 7?	2 and 7?	3 and 7?	4 and 7?
1 and 6?	2 and 6?	3 and 6?	4 and 6?
1 and 9?	2 and 9?	3 and 9?	4 and 9?
1 and 8?	2 and 8?	3 and 8?	4 and 8?
1 and 11?	2 and 11?	3 and 11?	4 and 11?
1 and 10?	2 and 10?	3 and 10?	4 and 10?
1 and 12?	2 and 12?	3 and 12?	4 and 12?

5 and 1?	6 and 1?	7 and 1?	8 and 1?
5 and 3?	6 and 3?	7 and 3?	8 and 3?
5 and 2?	6 and 2?	7 and 2?	8 and 2?
5 and 5?	6 and 5?	7 and 5?	8 and 5?
5 and 4?	6 and 4?	7 and 4?	8 and 4?
5 and 7?	6 and 7?	7 and 7?	8 and 7?
5 and 6?	6 and 6?	7 and 6?	8 and 6?
5 and 9?	6 and 9?	7 and 9?	8 and 9?
5 and 8?	6 and 8?	7 and 8?	8 and 8?
5 and 11?	6 and 11?	7 and 11?	8 and 11?
5 and 10?	6 and 10?	7 and 10?	8 and 10?
5 and 12?	6 and 12?	7 and 12?	8 and 12?

9 and 1?	10 and 1?	11 and 1?	12 and 1?
9 and 3?	10 and 3?	11 and 3?	12 and 3?
9 and 2?	10 and 2?	11 and 2?	12 and 2?
9 and 5?	10 and 5?	11 and 5?	12 and 5?
9 and 4?	10 and 4?	11 and 4?	12 and 4?
9 and 7?	10 and 7?	11 and 7?	12 and 7?
9 and 6?	10 and 6?	11 and 6?	12 and 6?
9 and 9?	10 and 9?	11 and 9?	12 and 9?
9 and 8?	10 and 8?	11 and 8?	12 and 8?
9 and 11?	10 and 11?	11 and 11?	12 and 11?
9 and 10?	10 and 10?	11 and 10?	12 and 10?
9 and 12?	10 and 12?	11 and 12?	12 and 12?

§ 23. What is the amount of

1, 3, and 2?	2, 7, and 3?	5, 3, and 4?	5, 5, and 7?
2, 3, and 3?	3, 6, and 4?	7, 6, and 6?	6, 5, and 8?
3, 4, and 5?	4, 2, and 5?	8, 4, and 2?	3, 8, and 7?
5, 1, and 2?	8, 3, and 2?	5, 9, and 3?	4, 6, and 9?
4, 2, and 6?	4, 6, and 5?	6, 7, and 4?	6, 3, and 8?
1, 7, and 4?	7, 3, and 4?	8, 1, and 7?	1, 9, and 7?
3, 2, and 5?	9, 4, and 5?	2, 4, and 6?	8, 4, and 6?
4, 5, and 3?	8, 5, and 3?	3, 5, and 9?	5, 4, and 7?
5, 7, and 2?	7, 8, and 2?	5, 4, and 3?	3, 5, and 7?
4, 9, and 3?	4, 6, and 7?	4, 2, and 7?	6, 8, and 3?
8, 2, and 4?	3, 9, and 2?	6, 4, and 5?	3, 7, and 4?
7, 3, and 7?	2, 9, and 6?	9, 3, and 4?	4, 8, and 5?
8, 5, and 4?	2, 5, and 8?	6, 8, and 7?	6, 7, and 4?
6, 2, and 4?	3, 6, and 7?	5, 6, and 8?	5, 8, and 6?
5, 3, and 5?	4, 5, and 8?	4, 6, and 8?	4, 9, and 6?

§ 24.—1. James has 3 cents in one pocket and 4 in another. How many in both?

2. George has 4 shillings and finds 3 more? How many has he now?

3. An orange costs 5 cents, a lemon 4 cents. How much will both cost?

4. Susan has 6 apples and Jane has 8. How many have both?

5. Jane has 6 oranges and Lucy has 9. How many have they both?

6. There are 7 cherries in one cluster and 8 in another. How many in both?

7. There are 6 scholars in one class and 9 in another. How many in both?

8. Edward has 8 cents and George 9. How many have both?

9. Joseph has 11 marbles and James 8 How many have both?

10. Samuel caught 8 fishes and Joseph 9. How many did both catch?

11. A boy having 10 marbles bought 5. How many did he then have?

12. Paid 9 dollars for a coat and 7 dollars for a pair of boots. How much for both?

13. Paid 11 dollars for a watch and 6 dollars for a chain. How much for both?

14. Paid 9 shillings for one book and 7 for another. How many shillings for both?

15. Paid 8 dollars for a coat and 5 dollars for a hat. How many dollars for both?

16. Paid 10 cents for a top and 9 cents for a ball. How many for both?

17. Paid 7 cents for a speller and 12 cents for a primer. How many for both?

18. Paid 9 cents for a reader and 8 cents for a table-book. How many for both?

19. James performed 11 sums and George 8. How many did both perform?

20. Jane spelled 12 words and missed 5. How many were pronounced to her?

21. Samuel spelled 8 words and missed 9. How many were pronounced to him?

22. Joseph performed 11 sums and Samuel 9. How many did both perform?

23. John has 5 apples and gave away 11. How many had he at first?

24. Sold 7 yards of calico and 11 yards of muslin. How many yards of both?

25. Sold 6 bushels of apples and 9 bushels of potatoes. How many bushels of both?

26. Bought 7 acres of land of A., 4 acres of B., and 5 acres of C. How many acres did I buy?

27. Paid 9 dollars for a coat, 5 dollars for a pair of pantaloons, and 3 dollars for a vest. How much for all?

28. Charles performed 7 sums on Monday, 8 sums on Tuesday, and 9 sums on Wednesday. How many in the three days?

29. James has 9 marbles, John 6, and Peter 8. How many marbles have they all?

30. What is the amount of 7, 8, 9, and 5?

NOTE. In adding the numbers of the following Table, *add the* 10s *only and place a cipher on the right of their sum.*

§ 25. ADDITION TABLE.—*Tens only.*

10+ 0= 10	20+ 0= 20	30+ 0= 30
10+ 10= 20	20+ 10= 30	30+ 10= 40
10+ 20= 30	20+ 20= 40	30+ 20= 50
10+ 30= 40	20+ 30= 50	30+ 30= 60
10+ 40= 50	20+ 40= 60	30+ 40= 70
10+ 50= 60	20+ 50= 70	30+ 50= 80
10+ 60= 70	20+ 60= 80	30+ 60= 90
10+ 70= 80	20+ 70= 90	30+ 70=100
10+ 80= 90	20+ 80=100	30+ 80=110
10+ 90=100	20+ 90=110	30+ 90=120
10+100=110	20+100=120	30+100=130
10+110=120	20+110=130	30+110=140
10+120=130	20+120=140	30+120=150
40+ 0= 40	50+ 0= 50	60+ 0= 60
40+ 10= 50	50+ 10= 60	60+ 10= 70
40+ 20= 60	50+ 20= 70	60+ 20= 80
40+ 30= 70	50+ 30= 80	60+ 30= 90
40+ 40= 80	50+ 40= 90	60+ 40=100
40+ 50= 90	50+ 50=100	60+ 50=110
40+ 60=100	50+ 60=110	60+ 60=120
40+ 70=110	50+ 70=120	60+ 70=130
40+ 80=120	50+ 80=130	60+ 80=140
40+ 90=130	50+ 90=140	60+ 90=150
40+100=140	50+100=150	60+100=160
40+110=150	50+110=160	60+110=170
40+120=160	50+120=170	60+120=180
70+ 0= 70	80+ 0= 80	90+ 0= 90
70+ 10= 80	80+ 10= 90	90+ 10=100
70+ 20= 90	80+ 20=100	90+ 20=110
70+ 30=100	80+ 30=110	90+ 30=120
70+ 40=110	80+ 40=120	90+ 40=130
70+ 50=120	80+ 50=130	90+ 50=140
70+ 60=130	80+ 60=140	90+ 60=150
70+ 70=140	80+ 70=150	90+ 70=160
70+ 80=150	80+ 80=160	90+ 80=170
70+ 90=160	80+ 90=170	90+ 90=180
70+100=170	80+100=180	90+100=190
70+110=180	80+110=190	90+110=200

§ **26.** What is the amount of

10, 30, and 20?	80, 70, and 50?	50, 30, and 80?
20, 40, and 50?	80, 10, and 90?	60, 70, and 40?
30, 60, and 20?	70, 70, and 20?	70, 90, and 20?
60, 30, and 70?	80, 40, and 90?	100, 80, and 10?
80, 10, and 20?	90, 30, and 50?	40, 50, and 20?
10, 90, and 50?	50, 70, and 20?	80, 30, and 10?
80, 10, and 70?	60, 10, and 80?	100, 60, and 30?
40, 90, and 20?	40, 50, and 110?	80, 70, and 50?
50, 80, and 70?	50, 60, and 120?	20, 30, and 90?
60, 20, and 30?	80, 70, and 90?	100, 60, and 110?
40, 50, and 80?	60, 70, and 100?	70, 80, and 50?
30, 90, and 40?	40, 50, and 90?	60, 10, and 100?
70, 80, and 30?	50, 40, and 80?	60, 70, and 50?
30, 90, and 70?	50, 40, and 90?	60, 10, and 120?
60, 80, and 70?	50, 90, and 80?	70, 30, and 20?
80, 90, and 70?	90, 30, and 110?	10, 60, and 90?
20, 90, and 90?	30, 80, and 50?	80, 70, and 50?

§ **27.** In combining the following tens, *add the figures only which occupy tens' place, then place a cipher on the right of the amount.*

1. James has 30 cents, and John 20. How many have both?

EXPLANATION. 30=3 tens, and 20=2 tens; and 3 tens +2 tens=5 tens, or 50. (§ 4.)

2. In a certain school there are 30 boys and 10 girls. How many scholars?

3. James has 30 marbles and John 20. How many have both?

4. If a horse cost 40 dollars, and a cow 20, how many will both cost?

5. In one basket there are 50 apples, and 40 in another. How many in both?

6. In one basket there are 60 apples, and 50 in another. How many in both?

7. Bought of one man 20 pounds of butter, of an-

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27. How combine tens?

other 30, and of a third 40. How many pounds did I buy?

8. John gave a poor man 50 cents, James gave him 40 cents, and Joseph 10 cents. How many did they all give him?

9. There are three sheepfolds; one of which contains 70 sheep, another 50, and the third 30. How many sheep in the three folds?

10. James has 30 cents, John 90 cents, and William 40 cents. How many have they all?

11. Of three rolls of ribbon, one contains 10 yards, another 40 yards, and another 60 yards. How many yards in the three rolls?

12. Having 70 dollars in my pocket, I borrowed 50 more, and collected 20 due me. How many had I then?

13.	How many are 10, 20, and 30?	60, 80, and 90?
14.	How many are 10, 30, and 50?	50, 70, and 30?
15.	How many are 20, 50, and 60?	40, 80, and 20?
16.	How many are 30, 70, and 50?	50, 70, and 70?
17.	How many are 70, 60, and 20?	30, 90, and 70?
18.	How many are 20, 90, and 50?	40, 80, and 90?
19.	How many are 30, 10, and 80?	50, 10, and 90?
20.	How many are 50, 80, and 30?	60, 20, and 80?
21.	How many are 90, 10, and 60?	30, 30, and 90?
22.	How many are 70, 40, and 50?	40, 70, and 70?
23.	How many are 60, 70, and 80?	50, 70, and 90?
24.	How many are 70, 80, and 90?	70, 60, and 70?

§ **28.** In adding the following tens and units, first add the tens, and then unite the units with the tens. (§ 7.) If more than one number of units are given, unite them into one number; then join the tens of this number, if any, to the previous tens.

1. A boy has in one of his vest pockets, 20 cents; in the other, 30 cents; and in a pantaloons pocket, 9 cents. How many cents has he?

EXPLANATION. 2 tens + 3 tens = 5 tens, or 50; and 5 tens, or 50 + 9 units = 59, *Ans.* (§ 7.)

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28. How combine tens and units?

2. How many are 20 cents, 50 cents, and 5 cents? 3. 30 cents, 50 cents, and 8 cents? 4. 40 cents, 70 cents, and 8 cents? 5. 30 cents, 50 cents, and 7 cents? 6. 80 cents, 60 cents, and 9 cents? 7. 50 cents, 70 cents, and 6 cents?

8. How many are 60, 30, and 7? 9. 20, 70, and 8? 10. 40, 90, and 3? 11. 70, 80, and 4? 12. 60, 90, and 7? 13. 50, 70, and 7?

14. If I buy 50 yards of ribbon of A, 70 yards of B., 6 yards of C., and 8 yards of D., how many yards do I buy?

EXPLANATION. 50=5 tens, and 70=7 tens; again, 6 units+8 units=14 units, or 1 ten and 4 units; then 5 tens+7 tens+1 ten+4 units=13 tens and 4 units=134, *Ans.*

15. If I buy a horse for 70 dollars, a saddle for 30 dollars, a harness for 9 dollars, and a bridle for 3 dollars, how much do the whole cost me?

16. How many are 10, 20, and 5? 5, 9, 8, and 80?
17. How many are 30, 40, and 7? 7, 7, 6, 50, and 60?
18. How many are 50, 60, and 1? 9, 5, 4, 40, and 80?
19. How many are 70, 80, and 4? 7, 6, 3, 90, and 10?
20. How many are 60, 50, and 9? 90, 8, 50, and 6?
21. How many are 60, 30, 7, and 5? 5, 80, 7, and 60?
22. How many are 50, 90, 9, and 5? 9, 7, 20, and 70?
23. How many are 60, 70, 8, and 7? 80, 70, 6, 3, and 4?
24. How many are 70, 50, 6, and 9? 50, 90, 7, and 8?
25. How many are 50, 90, 7, and 7? 60, 70, 9, and 1?
26. How many are 30, 70, 7, and 3? 8, 70, 9, and 90?

§ **29.** To add numbers larger than 10, and not consisting of 10s, resolve the given numbers into the tens and units of which they severally are composed; then unite the tens and units, as at § **28.**

1. What is the amount of 25, 33, and 15?

EXPLANATION. 25=2 tens and 5 units; 33=3 tens and 3 units; and 15=1 ten and 5 units; and 2 tens+3 tens+1 ten=6 tens; also, 5 units+3 units+5 units=13 units=1 ten and 3 units; and finally, 6 tens+1 ten+3 units=7 tens+3 units=73.

2. What is the amount of 15, 26, and 34?

3. What is the amount of 17, 63, and 81 ?
4. What is the amount of 21, 58, and 42 ?
5. What is the amount of 33, 44, and 55 ?
6. What is the amount of 28, 56, and 91 ?
7. What is the amount of 43, 59, and 84 ?
8. What is the amount of 52, 67, and 18 ?
9. What is the amount of 37, 48, and 63 ?
10. What is the amount of 18, 88, and 62 ?
11. What is the amount of 63, 91, and 35 ?
12. What is the amount of 18, 23, and 51 ?
13. What is the amount of 19, 36, and 76 ?
14. What is the amount of 26, 54, and 39 ?
15. What is the amount of 12, 62, and 72 ?
16. What is the amount of 33, 61, and 79 ?
17. What is the amount of 18, 27, and 67 ?
18. What is the amount of 28, 43, and 58 ?
19. What is the amount of 59, 64, and 81 ?
20. What is the amount of 73, 59, and 47 ?
21. What is the amount of 44, 62, and 85 ?
22. How many are 13, 85, and 97 ? 81, 56, and 43 ?
23. How many are 27, 69, and 67 ? 44, 55, and 66 ?
24. How many are 39, 58, and 79 ? 49, 57, and 73 ?
25. How many are 77, 83, and 59 ? 56, 83, and 47 ?
26. How many are 83, 59, and 27 ? 84, 39, and 66 ?
27. How many are 43, 76, and 56 ? 47, 81, and 72 ?
28. How many are 66, 43, and 87 ? 22, 59, and 68 ?
29. How many are 99, 21, and 38 ? 54, 62, and 37 ?
30. How many are 86, 68, and 45 ? 45, 73, and 96 ?

§ **30.**—1. How many are 20, 5, 7, 8, and 9 ?

NOTE. Resolve all numbers larger than 10 into the tens and units of which they are composed, before adding; then add the tens and units separately.

2. How many are 7, 27, 5, 6, and 8 ?
3. How many are 5, 6, 7, 3, and 45 ?
4. How many are 16, 8, 5, 4, and 36 ?
5. How many are 8, 5, 7, 6, and 21 ?
6. How many are 27, 6, 5, 4, and 11 ?
7. How many are 1, 5, 3, 7, and 77 ?

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30. Into what are numbers to be resolved, before adding ?

8. How many are 3, 46, 8, 7, and 9 ?
9. How many are 5, 7, 81, and 15 ?
10. How many are 18, 28, 1, 16, and 5 ?
11. How many are 71, 56, 7, 8, and 9 ?
12. How many are 81, 4, 17, 9, and 12 ?
13. How many are 15, 6, 9, 8, and 17 ?
14. How many are 9, 8, 7, 6, and 54 ?
15. How many are 16, 17, 8, 9, and 66 ?
16. How many are 5, 9, 7, 6, and 57 ?
17. How many are 9, 4, 3, 2, 1, and 85 ?
18. How many are 17, 28, 33, 6, and 9 ?
19. How many are 83, 57, 6, 5, 7, and 8 ?
20. How many are 17, 37, 5, 9, 6, and 7 ?
21. How many are 7, 6, 3, 2, and 30 ?
22. How many are 5, 4, 6, 5, and 33 ?
23. How many are 46, 8, 7, 5, and 22 ?
24. How many are 45, 8, 4, 12, and 13 ?
25. How many are 37, 4, 5, 7, and 6 ?
26. How many are 12, 13, 6, 8, and 4 ?
27. How many are 2, 4, 6, 8, and 88 ?
28. How many are 80, 8, 6, 4, and 2 ?
29. How many are 4, 6, 8, 7, and 81 ?
30. How many are 33, 7, 5, 9, 6, and 21 ?
31. How many are 42, 3, 5, 16, and 8 ?
32. How many are 63, 3, 5, 7, and 81 ?
33. How many are 11, 13, 6, 7, and 39 ?
34. How many are 3, 4, 5, 6, and 78 ?
35. How many are 18, 19, 4, 6, 8, and 1 ?
36. How many are 1, 77, 5, 6, 8, and 4 ?
37. How many are 3, 16, 29, 6, 7, and 9 ?
38. How many are 42, 51, 6, 7, 8, and 5 ?
39. How many are 39, 5, 6, 7, 81, and 7 ?

WRITTEN ARITHMETIC.

§ **31.** Arithmetic is said to be *written* when performed by the aid of the slate and pencil. Written arithmetic is employed more especially when the numbers are large.

31. When is arithmetic said to be written? When is written arithmetic employed?

SIMPLE ADDITION.

§ 32.—1. Addition consists in collecting several numbers into a single number, embracing the units, tens, hundreds, thousands, &c., of the several numbers.

2. The number obtained by adding several numbers together, is called the *amount*.

3. The addition of two or more numbers into one, is expressed by the sign +, called *plus*. The addition of 5, 19, and 8, is thus expressed: 5+19+8; and when added, the work is thus expressed: 5+19+8=32. This operation is thus read: 5 plus 19 are 24, and 24 plus 8 are 32. (§ 20.)

4. Only numbers expressing things of the *same kind*, can be joined together, so as to form but one number. If 7 represents a number of apples, and 8 a number of oranges, when added, their sum will express neither apples nor oranges; for, while it will be composed of 15 units, (1 ten and 5 units,) 7 of these units will still be apples, and 8 oranges. When two such numbers are joined, their sum simply expresses the *number of objects*, without any regard to their *kind*.

RULE FOR ADDITION.

Write the several numbers with units standing under units, tens under tens, hundreds under hundreds, &c., and add the several columns of figures separately, commencing with the right hand. In adding the units, if they amount to more than 10, *add the* 10*s to the column of* 10*s, setting down the units only; in adding the column of* 10*s, add the* 10*s of that column to the column of* 100*s; and in adding the column of hundreds, add the*

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32. In what does addition consist? What does the number obtained contain? What is the number obtained called? How is addition expressed? What is the sign called? How is an operation read? What numbers can be added? Illustrate. When such numbers are joined, what does their sum express? How write numbers for addition? How add the several columns? If the units amount to more than 10, what is done? What, if the tens amount to more than 10? And the hundreds to more than 10? &c. How write down the amount of each column? How write down the unit figure of each column?

10s to the column of 1000s; and so on, to the left-hand column, of which write down the whole amount.

NOTE. The unit figure of the amount of each column must be written down at the foot of the column, and *that one only*; except at the *left-hand* column, where the *whole amount* must be written down.

§ 33. What is the amount of 2543, 4285, 6456, and 3523?

Thou.	Hund.	Tens.	Units.
2	5	4	3
4	2	8	5
6	4	5	6
3	5	2	3
16	8	0	7

I first write down these numbers with units under units, tens under tens, hundreds under hundreds, &c., placing *units* first on the right, with *tens* next to units, *hundreds* next to tens, *thousands* next to hundreds, &c. I first add the column of units, and find that the figures of that column amount to 17=1 ten and 7 units. The 7 units I write down, and add the 1 ten to the column of tens. I next add the column of tens, and find that with the 1 ten brought from the column of units, it amounts to 20 tens, and as this makes just 2 hundred, and no remainder, I write down a cipher at the foot of the column of tens, and add 2 to the column of hundreds. Then adding the column of hundreds, I find it amounts to 18, including the 2 brought from the column of tens. Writing down the 8, I add the 1 ten=1 thousand, to the column of thousands, and adding that column, find the amount to be 16, the whole of which I write down, this being the last left-hand column.

§ 34.—1. By carefully studying the above example and its explanation, the nature of addition will be clearly understood. Each column it will be observed, is added separately, and the right-hand figure of the amount written down, while its left-hand figure is added to the next column. This is the invariable mode of procedure with each column but the last; of this the whole amount is written down.

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33. How write down the numbers? Which column add first? Which figure of the 17 write down, and which carry forward to the next column? The column of tens amounts to 20, which figure write down, and which carry forward? &c.

34. How is each column added? Which figure of the amount written down? What is done with the left-hand figure? Of which column is the whole amount written down?

2. The following exercise will familiarize the scholar with writing down the several amounts of the columns he adds up.

3. If a column of figures amounts to 17, which figure do you write down, and which add to the next column? Which write down, and which carry forward when the figures of a column amount to 25? to 33? to 18? to 47? to 59? to 27? to 35? to 46? to 54? to 63? to 71? to 89? to 54? to 96? to 84? to 66? to 28? If the whole amount of a column be only 7, what is done? if it be 3? if it be 5? if 7? if 6? if 4? if 9? if any figure of units only? Which is written down and which carried forward to the next column, if a column amounts to 50? to 60? to 87? to 96? to 83? to 90? to 95? to 76? to 84? to 79?

The scholar will now proceed to add the following sums:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
2345	4567	6435	6135	4567	3574
4632	5321	3542	3844	5678	8673

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
5465	4680	5678	4563	6658	5612
4538	1357	5867	6458	7743	3456
3462	9468	2956	4596	5564	7890

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
9876	6459	4685	6666	7777	5555
5432	3784	3765	7777	8888	9999
1089	5678	4268	8888	9999	8888

(19.)	(20.)	(21.)	(22.)	(23.)
8989	5678	24689	39567	45678
7878	6789	34674	45678	12645
6767	9868	95673	98764	63179

(24.)	(25.)	(26.)	(27.)	(28.)
56789	34653	46808	34756	34567
36571	27956	54327	45683	45964
45636	43637	45678	56394	37156
26547	36871	64361	67545	46271

(29.)	(30.)	(31.)	(32.)	(33.)
67890	56789	645567	756684	456678
70529	38754	43155	31564	56789
34715	48573	63478	23751	98756
63846	67569	59674	56340	87564
32593	42678	32263	67389	32105

(34.)	(35.)	(36.)	(37.)
563675	634567	834756	759635
657657	564633	543754	376375
476583	856734	465676	647568
375967	667788	678794	337594
438754	778844	109630	463742

(38.)	(39.)	(40.)	(41.)
665533	456789	875467	567890
337788	375696	375974	680246
567890	268752	695785	426873
445566	456374	467577	596847
593784	556974	345678	456752
339353	913957	354214	446687

(42.)	(43.)	(44.)	(45.)
356769	784694	756372	556677
475674	567435	573856	889900
356745	467578	467594	224466
656375	675789	563726	337788
786954	456785	463752	556677
875697	756386	543789	345678

(46.)	(47.)	(48.)	(49.)
347968	876678	573271	563742
745657	375912	375647	467538
457869	374568	545678	456789
107563	237594	675438	275645
456370	576971	234567	378956
256738	246812	356721	673065

(50.)	(51.)	(52.)	(53.)
463785	678904	6	358
576378	35946	87	45657
425927	4567	654	2358
386742	785	2738	546579
387569	69	46957	365667
667788	8	965738	583002

(54.)	(55.)	(56.)	(57.)
600006	777777	837564	893574
591073	666666	903675	7856
36744	888888	678649	86459
7658877	555555	377967	9,12345
766544	444444	456789	1234566
302070	999999	310952	389741
345678	876543	783646	875415

(58.)	(59.)	(60.)	(61.)
967345	878673	998877	123456
387569	566887	556688	654321
456832	467896	998756	987654
675786	338956	789568	241789
556677	795319	985676	674139
668866	864231	305097	441122
889989	967564	379518	889944
127581	194257	314253	763912

§ **35.** The following sums are to be added in the usual manner, and also by couplets, triplets, &c., as indicated by the brackets. The amounts of the first addition by couplets, triplets, &c., are again to be added in the same manner, till the whole amount is expressed in a single sum. The two results may then be compared, and if the work has been rightly performed, they will be found exactly to correspond. The object of the following sums is to secure extensive exercise in adding.

(1.)

35764				
45678	= 81442			
27563		=204644		
95689	=123202			
76887			=478252	
76456	=153343			
67398		=268608		
47867	=115265			
95673				=1023013
84567	=180240			
75679		=299884		
48965	=119644			
56758			=549761	
74695	=131453			
63756		=249877		
54668	=118424			
1023013				

The following sums may be added by triplets, as far as practicable, and also in the ordinary manner, and the results compared.

(2.)	(3.)	(4.)	(5.)
35768	89576	36856	56897
49674	95678	55387	71256
56978	42754	55874	34678
82567	39786	45678	67895
69568	58964	67594	76657
47639	47699	95678	34567
77669	31978	76657	87563
83594	42678	59886	42798
54695	59564	38756	95679
38789	46895	67568	67565
54689	37569	58689	43167
95638	45678	99786	74787
48675	77869	75968	86426
37958	44668	56789	45675
48769	78956	67896	36789
31153	93195	95674	97531
55331	91957	56767	38758
11355	91573	78989	87669

The following sums may be added by quadruplets.

(6.)	(7.)	(8.)	(9.)
68756	89578	38756	37856
42786	39567	67896	87869
67875	67695	84676	23785
41329	86747	75967	46689
75738	56879	39751	33779
45875	97567	86769	99887
64375	13456	43678	88776
56867	67895	95747	77665
95643	45738	36867	88575
38379	87654	75968	66554
56787	98765	32759	55443
56873	67890	98654	44332
87567	46875	73568	86789
56698	37569	75987	75632
56796	43756	46875	46789
37587	56789	87569	98756

(10.)	(11.)	(12.)	(13.)
895678	579345	875437	825967
537864	875648	675677	426795
357675	397586	459768	378789
467867	956428	387954	967546
355886	107469	499768	756789
875678	756954	975310	875679
596787	867432	875678	967569
567432	375675	432167	578568
678569	421098	783567	378967
595873	675469	437869	567898
678758	785695	781695	891234
906756	578108	327876	567890
876375	334455	673597	375912
756958	667788	756789	785467
776688	991122	446769	675846
856745	334455	376758	598756
987654	667788	912345	379876
875643	990011	678987	546889
765432	223344	564321	678689
645321	556677	468907	778899

(14.)	(15.)	(16.)	(17.)
578345	786945	756386	886644
775967	387564	967563	997755
659638	867559	586756	547163
456789	378196	789768	186756
378967	486876	321975	943787
581066	689753	687598	871678
778956	789564	567898	567834
432187	567875	371964	737856
590321	367856	987658	438975
753109	749687	378765	567843
678675	678175	674395	789567
438756	876596	835676	387246
745967	756783	567837	687564
875674	456738	876758	752786
367856	356789	673768	395678
275968	756867	886677	763475
917564	567867	597564	646874
368749	668893	568779	567898
546678	721098	378964	759643
759624	374567	559876	378756
538767	278918	328107	897568
486759	992018	917893	776754
956783	765438	567890	327867
576837	678912	901768	567868

NOTE. It will be remembered that the addition of these sums is to be effected both by finding the partial amounts as illustrated by operation 1, (§ 35,) and also by the ordinary direct method. The results will in all cases correspond, and thus form a test of the accuracy of the scholar's work, while at the same time it affords him extensive practice in the addition of numbers.

§ **36.** *Combinations by* 10. The process of adding may often be greatly simplified. It is obvious that when any number is to be increased by 10 or any number of 10s, its *unit* figure remains unaltered, while that occupying ten's place is increased by the given number of 10s. Thus, 67 increased by ten becomes 77; by 2 tens, 87; and by

36. By adding any number of 10s to a given number is the unit figure altered? How is the figure in ten's place affected? Illustrate.

3 tens, 97, &c. Hence, in adding, the practiced eye *will readily detect any two or more numbers whose combined units equal one or more* 10s, and add their amount by 10s.

The following sums will serve as an exercise of adding by 10s:

(1.)	(2.)	(3.)	(4.)
356745	875691	823567	778864
754365	235419	287543	332246
938471	987654	987654	123456
172639	123456	123456	987654
546756	759864	876543	234567
564354	351246	234567	876543
789164	859183	765432	917342
321946	251927	345678	193768
563764	345678	654321	817563
547346	765432	456789	293547
973171	546719	345673	756945
137939	564391	765437	354165
246813	789123	673128	982756
864297	321987	437982	128354
135798	456789	566771	876543
975312	654321	544339	234567
467596	957389	891234	398621
643514	153721	219876	712489
186491	666247	625412	542316
924619	423321	431375	503362

§ 37. *Combinations by* 9.—The left-hand figure, of any number composed of two figures, is made larger by 1, whenever that number is increased by 9, while the right-hand figure is diminished by 1. This is true in every case, except when the right-hand figure is 1. When this is the case, the right-hand figure becomes 0, and the left-hand one is increased by 1.

ILLUSTRATION. 27+9=36; 36+9=45; 45+9=54,

Hence what may be done?

37. When any number is increased by 9, how is the left-hand figure of that number affected? How the right-hand? What exception? Illustrate.

&c. Here it is obvious that the addition of 9 to any number is always productive of a uniform result. Hence, whenever the figures to be added can be formed into combinations of 9, the work is rendered easier by so combining them.

The following sums are given for the application of the above principle:

(1.)	(2.)	(3.)	(4.)
375463	567361	716456	123456
624536	432638	283543	876543
123456	711223	181712	282374
876543	288776	818287	717625
187916	456781	345678	564374
812083	543218	654321	435625
456861	123456	778811	817161
543138	876543	221188	182838
715611	272427	336644	513172
284388	727572	663355	486827
347954	182735	178257	567686
652045	817264	821742	432313
123456	571632	316185	556713
876543	428367	683814	443286
887711	713518	561781	181723
112288	286481	438218	818276
375463	567361	716456	123456
624536	432638	283543	876543
123456	711223	181712	282374
876543	288776	818287	717625
546171	356411	571268	567316
433018	441285	723721	422481
785615	516171	356106	281761
232232	242613	464712	546116

In this section, add by 9s constantly. A few of the lower figures in each column do not add by 9s. This is an arrangement necessary to prevent the amounts of several columns being uniformly the same. Make these sums a class exercise upon the blackboard—the whole class adding audibly, and in uniform movement.

(5.)	(6.)	(7.)	(8.)
337189	734618	371653	741312
662810	265381	628346	258687
731122	345678	181607	511632
268877	654321	818392	488367
445671	713716	345613	133567
554328	286283	654386	866432
716151	567181	734371	711334
283848	432818	265628	288665
716980	713547	717273	171918
283019	286452	282726	828081
776644	234567	467856	673161
223355	765432	532143	326838
861354	817191	376757	123671
138645	182808	623242	876328
237164	671761	123456	171781
762835	328238	876543	828218
337189	734618	371563	312345
662810	265381	628436	687654
731122	345678	181607	781160
268877	654321	818392	218839
445671	713716	347613	718112
554328	286283	652386	281887
456241	361362	058070	212345
543828	628435	821828	685655
140710	201003	706376	560360
508376	586762	182401	218476

§ 38.—1. *Combinations by* 8.—The right-hand figure of any number consisting of two figures becomes less by 2, whenever that number is increased by 8, unless the right-hand figure be 1 which then becomes 9; while if any change take place in the left-hand figure, it becomes greater by 1.

2. A change always does take place in the left-hand figure whenever the right-hand figure is greater than 1.

38. When a number consisting of two figures is increased by 8, what change takes place in the right-hand figure? What in the left? When does a change take place in the left-hand figure?

Add the following combinations of 8.

(1.)	(2.)	(3.)	(4.)
376456	765432	811356	167345
512432	123456	677532	721543
776553	567183	753321	532731
112336	321705	135567	356157
703403	278167	283153	141516
185485	610721	605735	747372
617152	321615	117634	457681
271736	567273	771254	431207
334761	334455	351762	345678
554127	554433	537126	543210
137582	711612	537381	715131
751306	177276	351507	173757
517633	313671	676462	345567
371255	575217	212426	543321
417653	112233	713157	706152
471235	776655	175731	182736
263457	226655	123456	765426
625431	662233	765432	123462
130284	170272	667744	223355
758604	718616	221144	665533
723278	345660	566766	511648
165610	543228	322122	377240
165618	343524	372522	357246
622597	245573	616344	531642
531714	478217	942331	670147
146062	255632	012213	218741

§ 39.—1. Combinations by numbers less than 8 are perhaps undesirable. The following, however, consisting of numbers above 10, are worthy of attention.

2. *Combinations by* 11.—Each of the figures of any number of two figures, is made larger by 1, when that number is increased by 11, except when the right-hand figure is 9: in which case, the 9 becomes 0 and the left-hand figure is increased by 2.

39. How is each figure of any number of two figures affected when increased by 11? What exception? How then affected?

Add the following combinations of 11.

(1.)	(2.)	(3.)	(4.)
567894	987856	667788	567845
654327	234365	554433	654376
456789	785428	376645	842768
765432	436793	846576	379453
846743	876543	593745	246864
375478	345678	628476	975357
262729	827262	837483	866789
959492	394959	384738	355432
356873	637462	567874	246885
865348	584759	654347	975336
539286	234567	887766	857788
462935	987654	334455	364433
996877	765432	774666	747890
203163	376343	432709	443967

§ **40.** *Combinations by* 12.—When any number of two figures is increased by 12, the left-hand figure is increased by 1, and the right-hand one by 2. The only exceptions are, when the right-hand figure is 8, or 9, in which case this figure becomes 0 or 1, while the left-hand figure is increased by 2.

(1.)	(2.)	(3.)	(4.)
446748	443358	834765	975337
886584	889974	498567	357995
339765	665544	844335	974743
993567	667788	488997	358589
339765	765544	876335	944743
993567	567788	456997	388589
993756	987654	675678	456789
339576	345678	657654	876543
556677	876543	979893	567899
776655	456789	353439	765433
887766	567897	889977	678999
445566	765435	443355	654333
937543	898759	882536	769988
342946	460647	505577	547582

40. How is the left-hand figure of any number affected when that number is increased by 12? How the right-hand? What exceptions? How is it then?

§ **41.** *Combinations by* 8s, 9s, and 10s.—It will be observed that in all these combinations the figures taken by couplets amount to one of these numbers. (See § **36**, **37**, **38**.)

(1.)	(2.)	(3.)	(4.)
543672	785645	787164	556677
345218	325465	323935	333333
678567	453784	427846	789167
321432	435104	571153	221833
546787	567537	866785	567893
564323	432462	233225	443217
246756	754378	774466	897347
642132	245621	325634	112763
573456	781567	783645	345778
426543	309543	125255	764331
756784	676435	395674	567817
354326	324664	615436	443292
845672	778856	778811	778196
265438	222232	222288	231903
675934	167847	335566	678912
435176	833262	764534	331198
427564	756789	279567	775637
572435	244321	721441	235472
463752	867532	667734	556677
425136	243368	343266	453423
913785	756935	789813	756323
197325	243165	111277	342766
756789	378789	825967	976310
354321	522321	275132	134799
387954	675677	421098	867432
503136	435433	479812	243556
107460	897586	678569	596787
992428	213524	321430	514323
875678	355886	357675	815678
125210	655224	531214	194321

§ **42.** *Combinations by* 9s, 10s, *and* 11s.—Let the work be strictly performed by combinations. (See § **36**, **37**, **39**.)

(1.)	(2.)	(3.)	(4.)
637456	375678	756384	867567
362543	624321	354726	333444
817564	865643	567890	567895
283645	246378	444329	454324
728394	475961	861567	389756
473626	646238	338442	821345
887569	834567	753186	987654
324532	366654	347815	223346
878179	834696	896754	789876
132921	377423	314245	212333
769656	663686	789345	674768
332445	448515	311775	545453
876597	786329	897867	834567
225414	335891	323232	367433
763696	963456	579864	912345
246523	238645	431157	188666
638256	668754	863475	758673
372844	353456	358734	363548

(5.)	(6.)	(7.)	(8.)
978763	667788	864376	456789
232448	334422	347643	545432
967437	667781	915456	667783
234774	344318	284654	454336
876646	567890	978766	747384
346363	454119	227435	254635
786798	637456	123456	776644
338411	584565	987543	445465
867896	887766	234567	567891
343225	323454	885434	443319
467875	786763	546567	869145
639846	233458	465643	331954
789087	875678	893456	987543
330934	334441	317544	233456
123456	678934	776655	789564
898854	333276	433444	431445
456789	813467	875463	789123
743332	398544	346758	231876

§ **43.** *Combinations of* 9s, 10s, 11s, *and* 12s. (§ **36, 37, 39, 40.**)—These references should be carefully studied before performing the sums.

(1.)	(2.)	(3.)	(4.)
875673	891675	987543	123486
345649	319457	344668	998545
789223	876198	789123	781967
433889	445934	341896	548143
769642	238418	675654	873845
443578	963894	345567	456367
678922	867639	875466	345678
453188	454693	336845	876634
789346	789128	789678	987563
531884	431983	542342	233539
223456	223459	667788	168864
898656	896873	554343	954146
786475	756434	775675	818357
335847	465678	546546	483865
987654	375673	837564	678346
134657	745638	374668	453865

(5.)	(6.)	(7.)	(8.)
786347	867643	846396	345678
546845	364478	465634	667453
437675	675674	756783	789654
694354	644648	464537	533467
876543	759638	867567	875676
346569	551563	454565	437445
191827	875437	678134	876543
939384	356665	544996	445566
678915	785639	467569	375675
454386	547693	565443	954455
785643	786498	656677	867566
446678	345632	446345	454463
964759	868156	786567	378645
337463	363954	543465	854656
864678	786954	789167	786759
467554	545356	543935	346571

§ 44. In adding the following sums, care should be taken to unite them by combinations as far as practicable. The labor of adding will be much diminished, when this process is once made familiar.

(1.)	(2.)	(3.)	(4.)
678564	345678	678956	678675
347564	567896	786546	567856
637156	678567	875107	467786
694367	345678	836795	637895
667788	678678	437896	667788
446756	456789	546783	991122
345678	346784	598376	334455
455789	378917	456789	667788
678564	986356	988654	901234
367180	786789	327565	567890
786756	786798	971836	478643
987654	468357	186467	678954
321067	987642	336781	378645
898765	671986	466786	678676
432168	786781	785678	567839
567891	426808	336981	678546

(5.)	(6.)	(7.)	(8.)
684673	787695	876398	678367
567891	467679	954676	863845
346789	567638	875677	786785
475678	786754	786394	467863
678745	456879	806743	678163
567867	548697	786756	789109
786467	967876	867865	813461
976767	867567	786756	786967
872637	378716	967867	467869
866746	956786	410763	678346
567375	764376	456789	961075
678997	769378	567890	467845
374769	123405	678901	568716
786369	234560	789012	735678
817636	304567	899123	468753
756309	450708	901234	786759

BUSINESS APPLICATION OF ADDITION.

§ **45.** Addition is employed in business transactions, to find the total value, weight, measure, or other numerical valuation of any specified number of objects, each object possessing its own specific value.

In writing down the numbers to be added into one, especial care must be taken *to place units under units, tens under tens, hundreds under hundreds, &c.*

1. If an orchard contain 384 apple-trees, 263 pear-trees, 782 peach-trees, and 179 plum-trees, how many trees are there in the orchard?

OPERATION.

3 8 4 = apple.
2 6 3 = pear.
7 8 2 = peach.
1 7 9 = plum.

1 6 0 8 = amount of all the trees in the orchard.

Just such is the operation in every case;—*we take the numbers expressing the different objects, and write them under each other, and then commencing with the right-hand column, we add the given numbers together, so as to make a single number express all the units, tens, &c., of the several given numbers.*

2. A bookseller has 1678 geographies, 8789 histories, 1378 grammars, 6789 arithmetics, 3796 testaments, and 8793 miscellaneous books. How many books has he in his store? *Ans.* 31223 books.

3. In the year 1837 there emigrated to America from England, 36796 men, women, and children; from Scotland, 3524; and from Ireland, 56334. How many came to this country from those three countries? *Ans.* 96654.

4. In the year 1842 the value of the exports from the state of Maine was $1018269; from New Hampshire, $20979; from Vermont, $305150; from Massachusetts, $10186261; from Connecticut, $518210; and from Rhode

45. For what is addition employed? What care must be taken? What do we write down in every case of addition? Where do we commence adding? What do we do?

Island, $206989. What was the value of the exports of these six New England states for that year?

Ans. $12255858.

Be careful in writing down these sums to place units under units, tens under tens, &c.

5. The value of imports of the same states for the same year was, of Maine, $628762; of New Hampshire, $23921; of Vermont, $404617; of Massachusetts, $16513858; of Connecticut, $277072; and of Rhode Island, $274534. What was the value of all the imports of these states?

Ans. $18122764.

6. The amount of shipping for the same states during the same year, was for Maine, 281930 tons; New Hampshire, 23921 tons; Vermont, 4343 tons; Massachusetts, 494895; Connecticut, 67749; and Rhode Island, 47243. What was their total tonnage? *Ans.* 920081 tons.

7. The exports for the same year for New York, were $34264080; for New Jersey, $16076; for Pennsylvania, $6820145; for Delaware, 37001; Maryland, $5768768; District of Columbia, $753923; Virginia, $4776220. What was the united value of exports from these states?

Ans. $52438213.

8. The imports for the same states and year, were for New York, $60440750; for New Jersey, $19209; for Pennsylvania, $8464882; for Delaware, $802; Maryland, $4910746; District of Columbia, $119852; and Virginia, $545085. What was the united value of the imports of these states? *Ans.* $74501326.

9. The shipping for these states was, for N. York, 518133 tons; New Jersey, 60742 tons; Pennsylvania, 113569 tons; Delaware, 10396 tons; Maryland, 106856 tons; District of Columbia, 17711 tons; Virginia, 47536 tons. What was their total tonnage? *Ans.* 874943 tons.

10. The exports for North Carolina, for the year 1842, amounted to the sum of $387484; S. Carolina, $10036769; Georgia, $6862959; Alabama, $12854694; Louisiana,

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What are imports? (See dictionary.) What are exports? (See dictionary.)

$34236936; Ohio, $991954; Michigan, $162229; Florida, 1858850. Required the total value of their exports.

Ans. $67391875.

11. Required the total amount of imports for the same states, that for North Carolina being $252532; for South Carolina, $2058870; for Georgia, $491428; for Alabama, $574651; Louisiana, $10673190; Ohio, $4915; Michigan, $138610; Florida, $190728. *Ans.* $14384924.

12. Required the total amount of tonnage for the same states, that of North Carolina being 31682 tons; for South Carolina, 23469 tons; for Georgia, 16536 tons; Alabama, 14577 tons; Louisiana, 144128 tons; Ohio, 24830 tons; Michigan, 12323 tons; Florida, 7288 tons.

Ans. 274833 tons.

13. What was the total value of the exports from all the states for the year 1842? (Unite the answers of sums 4, 7, and 10.) *Ans.* $132085946. What the total value of imports? (Sums 5, 8, 11.) *Ans.* $107009014. Also, their total amount of tonnage? (Sums 6, 9, 12.)

Ans. 2069857 tons.

14. The population of Maine in 1840, was 502000; of New Hampshire, 284574; of Vermont, 292000; of Massachusetts, 737699; of Rhode Island, 108830; and of Connecticut, 309978. Required the total population of all these states. *Ans.* 2235081.

15. At the same date the population of New York was 2428921; New Jersey, 373306; Pennsylvania, 1724023; Delaware, 78065; Maryland, 470000; Virginia, 1239797; District of Columbia, 43712. Required the total population of these states. *Ans.* 6357824.

16. Required the total population of the following states at the same date, that of each being as follows:—viz., of North Carolina, 753419; of South Carolina, 594398; Georgia, 691392; of Alabama, 590756; of Mississippi. 375651; Louisiana, 352411; Arkansas, 97574; of Tennessee, 829210; and of Kentucky, 779828.

Ans. 5064639.

17. The total population of the following states at the above date is also required; that of Ohio being 1519464; of Michigan, 212267; of Indiana, 685866; of Illinois,

476183; of Missouri, 339702; of Florida, 54477; of Wisconsin, 30945; and of Iowa, 43112.

Ans. 3362016.

18. What is the total population of all the states mentioned in the four preceding sums? *Ans.* 17019560

19. How many square miles are there in the New England states, that of Maine being 33000; New Hampshire, 9500; Vermont, 10000; Massachusetts, 7800; Rhode Island, 1451; and Connecticut, 4800?

Ans. 66551.

20. How many square miles in the following states, there being in New York, 46220; New Jersey, 7948; Pennsylvania, 46215; Delaware, 2068; Maryland, 10755; and Virginia, 65700? *Ans.* 178906.

21. What is the united extent of the following states; their individual extent being as follows:—Michigan, 60537 square miles; Ohio, 40500; Indiana, 35626; Kentucky, 40023; Tennessee, 41752; Arkansas, 54617; Missouri, 70050; Illinois, 56506; Iowa, 173786; and Wisconsin, 92930? *Ans.* 666327.

22. There are in the libraries of Bowdoin College, 25590 volumes; in the libraries of Waterville College, 8,100 volumes; in those of Dartmouth College, 16500 volumes; of Vermont University, 10000 volumes; of Middlebury College, 7054 volumes; of Norwich University, 1032 volumes; of Harvard University, 82000 volumes; of Williams' College, 9643 volumes; of Amherst College, 16000 volumes; College of the Holy Cross, 4220 volumes; of Brown's University, 27520 volumes; of Yale College, 47700 volumes; of Trinity College, 9000 volumes; and of the Wesleyan University, 11123. These colleges are all located in the New England states. How many volumes are there then in all the colleges of New England? *Ans.* 275482.

23. Maine has a school fund of $350000; Massachusetts, of $850767; Rhode Island, of $51300; Connecti-

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Can the scholar ascertain in which of the New England states each of these colleges is situated? What is a school fund? *Ans.* Money drawing interest, which interest is appropriated to the support of common schools. The character ($) is placed before figures when they represent dollars.

cut, of $2077641; New York, of $6491803; New Jersey, of $369278; Delaware, of $225000; Virginia, of $1488261; Georgia, $262300; Alabama, $1215381; Tennessee, $1346068; Kentucky, $1221819; Ohio, $1566931; Michigan, $500000; Indiana, $2195149; Missouri, $575668; and Iowa, $132909. The other states have no school fund. What is the amount of school funds owned by these several states? *Ans.* $20920275.

24. How much Banking Capital has the State of New Hampshire, the following being the capital belonging to the several Banks of the State, viz.: Amoskeag Bank, $100000; Ashuelot Bank, $100000; Belknap County Bank, $50000; Cheshire Bank, $100000; Claremont Bank, $60000; Connecticut River Bank, $90000; Derry Bank, $20000; Dover Bank, $75000; Granite Bank, $100000; Great Falls Bank, $100000; Lancaster Bank, $50000; Lebanon Bank, $100000; Manchester Bank, $100000; Mechanics' Bank, $100000; Merrimack Co. Bank, $80000; Mechanic's and Trader's Bank, $110000; New Ipswich Bank, $100000; Nashua Bank, $100000; Piscataqua Exchange Bank, $200000; Rochester Bank, $100000; Rockingham Bank, $143000; Strafford Bank, $100000; and Winchester Bank $100000? *Ans.* $2178000.

25. There were exported from the United States during the year 1847, products of the sea, amounting to the following sums, viz.: dried codfish, $669629; pickled fish of various kinds, $136221; whale and fish oils, $1070659; spermaceti oil, $738456; whalebone, $671601; and spermaceti candles, $191467. What was the total value of these products? *Ans.* $3478033.

26. The following are the exports of the sea from the United States for the year 1848, viz.: dried fish, $609482; pickled fish, $109315; whale and fish oils, $552388; spermaceti oil, $208832; whalebone, $314107; spermaceti candles, $186839. Required the total amount of these exports. *Ans.* $1980963.

27. The following sums show the exports of the forest during the year 1847, viz.: of skins and furs, $747145; of ginseng, $64466; of staves, shingles, boards, and hewn timber, $1849911; of other lumber, $342781; of masts

and spars, $23270; of bark and dye-wood, $95355; of wooden manufactures, $1495924; of naval stores, tar, pitch, rosin, and turpentine, $759221; of ashes, pot and pearl, $618000. Required the total value of these exports.

Ans. $5996073.

28. The following sums show the value of the exports of the forest for the year 1848, viz.: of skins and furs, $607780; ginseng, $162647; staves, shingles, boards, and hewn timber, $2429863; other lumber, $283433; masts and spars, $129760; bark and dye-wood, $184126; wooden manufactures, $2042694; tar, pitch, rosin, and turpentine, $752303; pot and pearl ashes, $466477. What is the total value of the exports of the forest for the year 1848?

Ans. $7059083.

29. Exports of animal products for the year 1847, viz.: of beef, tallow, hides, and horned cattle, $2434003; of butter and cheese, $1741770; pork, bacon, lard, and live hogs, $6630842; horses and mules, $277359; sheep, $29100; and wool, $89460. To what did the exports of animal products amount?

Ans. $11202534.

30. The exports of animal products for the year 1848 were as follows, viz.: beef, tallow, hides, and horned cattle, $1905341; butter and cheese, $1361668; pork, bacon, lard, and live hogs, $9003272; horses and mules, $190295; sheep, $20823; wool, $57497. Required the total value.

Ans. $12538896.

31. The following are the vegetable exports for the year 1847, viz.: wheat, $6049350; flour, $26133811; Indian corn, $14395212; Indian meal, $4301334; rye meal, $225502; rye, oats, and other small grain and pulse, $1600962; ship bread, $556266; potatoes, $109062; apples, $92961; rice, $3605896. Required the total value of vegetables exported that year.

Ans. $57070356.

32. The same exports for the year 1848 were as follows: wheat, $2669175; flour, $13194109; Indian corn, $3837483; Indian meal, $1807601; rye meal, $174566; rye, oats, and other small grain, $376572; ship bread, $619096; potatoes, $86277; apples, $88944; rice, $2331824. Required the total value of vegetable exports for the year 1848.

Ans. $25185647.

33. What is the value of the following vegetable products exported during the year 1847, viz.: tobacco, $7242086; cotton, $53415848; flax-seed, $1346; hops, $150654; brown sugar, $25483; and indigo, $10?

Ans. $60835427.

34. The exports of the same articles for the year 1848 were as follows: tobacco, $7551122; cotton, $61998294; hemp, $27657; flax-seed, $1584; hops, $17671; brown sugar, $8891; indigo, $1100. Required their total value.

Ans. $69606319.

35. Required the total value of the following amounts of manufactured articles exported in the year 1847, viz.: soap and tallow candles, $606798; leather boots and shoes, $243816; household furniture, $225700; coaches and carriages, $75369; hats, $59536; saddlery, $13102; wax, $161527; spirits from grain, $67781; beer, ale, porter, and cider, $68114; snuff and tobacco, $658950; linseed oil and spirits of turpentine, $498110; and cordage, $27054.

Ans. $2705857.

36. The following is the value of the same articles exported in 1848. Required their total value. Soap and tallow candles, $670223; leather boots and shoes, $194095; household furniture, $297358; coaches and carriages, $89963; hats, $55493; saddlery, $27435; wax, $134577; spirits from grain, $90957; beer, ale, porter, and cider, $78071; snuff and tobacco, $568345; linseed oil and spirits of turpentine, $331404; and cordage, $29911.

Ans. $2567832.

37. What is the total value of the following articles exported during the year 1847, viz.: pig and bar iron, and nails, $168817; iron castings, $68889; all manufactures of iron, $929778; spirits from molasses, $293609; refined sugar, $124824; chocolate, $1653; gunpowder, $88397; copper and brass, $64980; and medicinal drugs, $165793?

Ans. $1906740.

38. The following are the several values of the same articles exported during the year 1848, viz.: pig and bar iron, and nails, $154036; iron castings, $83188; iron manufactures, $1022408; refined sugar, $253900; chocolate, $2207; gunpowder, $125263; copper and brass,

$61468; spirits from molasses, $269467; medicinal drugs, $210581. Required the total value. *Ans.* $2182518.

39. During the year 1847, there were exported the following manufactured articles, viz.: printed and colored cotton fabrics, $281320; white do., $3345902; nankeen, $8794; twist, yarn, and thread, $108132; other manufactures of cotton, $338375; also manufactures of flax and hemp, $5305; cloth and thread, $477; wearing apparel, $47101; combs and buttons, $17026; brushes, $2967. Required their total value. *Ans.* $4155399.

40. The same articles were exported during the year 1848, to the following amounts. Printed and colored cotton fabrics, $351169; white cotton fabrics, $4866559; nankeen, $2365; twist, yarn, and thread, $170633; other manufactures of cotton, $327479; manufactures of flax and hemp, $6218; cloth and thread, $495; wearing apparel, $574834; combs and buttons, $16461; brushes, $2160. Required the total value. *Ans.* $6318373.

41. The following articles are enumerated among the articles exported during the year 1847, with their several values, viz.: billiard tables and apparatus, $615; umbrellas and parasols, $2150; leather and morocco skins, $29856; fire-engines and apparatus, $3443; printing presses and type, $17431; musical instruments, $16997; books and maps, $44751; paper and stationery, $88731; paints and varnish, $54115. Required their total value.

Ans. $258089.

42. The same articles were exported during the year 1848 to the following amounts, viz.: billiard tables, $12; umbrellas and parasols, $2916; leather and morocco skins, $16483; fire-engines and apparatus, $7686; printing presses and type, $30403; musical instruments, $88508; books and maps, $75193; paper and stationery, $78507; paints and varnish, $50739. Required their total value.

Ans. $350447.

43. The value of exports for the year 1847 of the following articles, was: of vinegar, $9526; earthen and stone ware, $4758; manufactures of glass, $71155; tin, $6363; pewter and lead, $13694; marble and stone, $11220; gold and silver, and gold-leaf, $4268; gold and sil-

ver coin, $62620; artificial flowers and jewelry, $3126; molasses, $26959. What was their total value?

Ans. $213689.

44. The same articles were exported during the year 1848 to the following amounts, viz.: vinegar, $13920; earthen and stone ware, $8512; glass manufactures, $76007; tin, $12353; pewter and lead, $7739; marble and stone, $22466; gold and silver, and gold-leaf, $6241; gold and silver coin, $2700412; artificial flowers and jewelry, $11217; molasses, $5563. Required their total value.

Ans. $2864430.

45. The value of exports for the year 1847 of the following articles, was: of trunks, $5270; brick and lime, $17623; salt, $42333; lead, $124081; various manufactured articles, $1108984; other articles, $1199276; government stores to the army from New York, $326800. Required their total value. *Ans.* $2824367.

46. During the year 1848, the exports of the same articles, were: of trunks, $6126; brick and lime, $24174; salt, $73274; lead, $84278; coal, $47112; ice, $75547; various manufactured articles, $1157828; other articles, $851383. Required their total value. *Ans.* $2299722.

47. Required the total value of exports for the year 1847. (Add together the answers of sums 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, and 45.) *Ans.* $150646564.

SUBTRACTION.

§ **46.**—1. Subtraction consists in taking one number from another number so as to find the difference between them.

2. The subtraction of one number from another is expressed by a short *horizontal line* (—) placed between the two numbers, and implying that the number placed on the right is to be taken from the one on the left;—thus the subtraction of 6 from 13 is expressed as follows: 13—6=7.

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46. What is subtraction? How is subtraction expressed? What *does it imply?*

§ 47. SUBTRACTION TABLE.—*Units only.*

1 taken	2 taken	3 taken	4 taken
from 1= 0	from 2= 0	from 3= 0	from 4= 0
2= 1	3= 1	4= 1	5= 1
3= 2	4= 2	5= 2	6= 2
4= 3	5= 3	6= 3	7= 3
5= 4	6= 4	7= 4	8= 4
6= 5	7= 5	8= 5	9= 5
7= 6	8= 6	9= 6	10= 6
8= 7	9= 7	10= 7	11= 7
9= 8	10= 8	11= 8	12= 8
10= 9	11= 9	12= 9	13= 9
11=10	12=10	13=10	14=10

5 taken	6 taken	7 taken	8 taken
from 5= 0	from 6= 0	from 7= 0	from 8= 0
6= 1	7= 1	8= 1	9= 1
7= 2	8= 2	9= 2	10= 2
8= 3	9= 3	10= 3	11= 3
9= 4	10= 4	11= 4	12= 4
10= 5	11= 5	12= 5	13= 5
11= 6	12= 6	13= 6	14= 6
12= 7	13= 7	14= 7	15= 7
13= 8	14= 8	15= 8	16= 8
14= 9	15= 9	16= 9	17= 9
15=10	16=10	17=10	18=10

9 taken	10 taken	11 taken	12 taken
from 9= 0	from 10= 0	from 11= 0	from 12= 0
10= 1	11= 1	12= 1	13= 1
11= 2	12= 2	13= 2	14= 2
12= 3	13= 3	14= 3	15= 3
13= 4	14= 4	15= 4	16= 4
14= 5	15= 5	16= 5	17= 5
15= 6	16= 6	17= 6	18= 6
16= 7	17= 7	18= 7	19= 7
17= 8	18= 8	19= 8	20= 8
18= 9	19= 9	20= 9	21= 9
19=10	20=10	21=10	22=10
20=11	21=11	22=11	23=11

§ 48. MENTAL EXERCISES IN SUBTRACTION.

1 from 1 = what?	5 from 5 = what?	9 from 9 = what?
1 " 3 = "	5 " 7 = "	9 " 11 = "
1 " 2 = "	5 " 6 = "	9 " 10 = "
1 " 5 = "	5 " 9 = "	9 " 13 = "
1 " 4 = "	5 " 8 = "	9 " 12 = "
1 " 7 = "	5 " 11 = "	9 " 15 = "
1 " 6 = "	5 " 10 = "	9 " 14 = "
1 " 9 = "	5 " 13 = "	9 " 17 = "
1 " 8 = "	5 " 12 = "	9 " 16 = "

2 from 2 = what?	6 from 6 = what?	10 from 10 = what?
2 " 4 = "	6 " 8 = "	10 " 12 = "
2 " 3 = "	6 " 7 = "	10 " 11 = "
2 " 6 = "	6 " 10 = "	10 " 14 = "
2 " 5 = "	6 " 9 = "	10 " 13 = "
2 " 8 = "	6 " 12 = "	10 " 16 = "
2 " 7 = "	6 " 11 = "	10 " 15 = "
2 " 10 = "	6 " 14 = "	10 " 18 = "
2 " 9 = "	6 " 13 = "	10 " 17 = "

3 from 3 = what?	7 from 7 = what?	11 from 11 = what?
3 " 5 = "	7 " 9 = "	11 " 13 = "
3 " 4 = "	7 " 8 = "	11 " 12 = "
3 " 7 = "	7 " 11 = "	11 " 15 = "
3 " 6 = "	7 " 10 = "	11 " 14 = "
3 " 9 = "	7 " 13 = "	11 " 17 = "
3 " 8 = "	7 " 12 = "	11 " 16 = "
3 " 11 = "	7 " 15 = "	11 " 19 = "
3 " 10 = "	7 " 14 = "	11 " 18 = "

4 from 4 = what?	8 from 8 = what?	12 from 12 = what?
4 " 6 = "	8 " 10 = "	12 " 14 = "
4 " 5 = "	8 " 9 = "	12 " 13 = "
4 " 8 = "	8 " 12 = "	12 " 16 = "
4 " 7 = "	8 " 11 = "	12 " 15 = "
4 " 10 = "	8 " 14 = "	12 " 18 = "
4 " 9 = "	8 " 13 = "	12 " 17 = "
4 " 12 = "	8 " 16 = "	12 " 20 = "
4 " 11 = "	8 " 15 = "	12 " 19 = "

§ 49.—1. Mary had 5 books, and gave away 2. How many had she left?

2. James had 6 apples, and gave away 3. How many had he left?

3. Susan had 7 oranges, and gave away 4. How many had she left?

4. Robert had 9 pennies, and spent 6 of them. How many had he left?

5. William had 11 cents, and gave 7 for a book. How many had he left?

6. John had 13 marbles, and lost 6 of them. How many had he left?

7. Robert picked 11 apples from a branch of an apple-tree containing 16 apples. How many did he leave?

8. James is 17 years old, and Samuel 8. How many years is James older than Samuel?

9. A man bought a cow for 12 dollars, and sold her for 18. How many dollars did he gain?

10. A man having 17 sheep sold 7. How many had he left?

11. A boy having 21 cents spent 12. How many had he left?

12. Paid $17 for a coat, and $8 for a pair of pantaloons. How many more dollars did I pay for my coat than for my pantaloons?

13. A man bought 3 barrels of flour for $18, and sold one of them for $7. What did the two remaining barrels cost him?

14. 15 less 7 are how many? 24 less 12 are how many?
15. 13 less 8 are how many? 18 less 9 are how many?
16. 17 less 9 are how many? 19 less 7 are how many?
17. 19 less 11 are how many? 17 less 8 are how many?
18. 15 less 7 are how many? 16 less 4 are how many?
19. 21 less 9 are how many? 13 less 8 are how many?
20. 22 less 11 are how many? 11 less 8 are how many?
21. 18 less 12 are how many? 20 less 11 are how many?

§ 50. In subtracting the following 10s, *subtract the figure occupying the tens' place only, and write a cipher on the right of the result.*

1. James having 40 cents, spent 20 cents. How many had he left?

EXPLANATION. 40=4 tens, and 20=2 tens; and 4 tens −2 tens=2 tens=20.

2. 30−10 are how many?	70−20 are how many?
3. 50−30 are how many?	60−40 are how many?
4. 40−10 are how many?	90−50 are how many?
5. 70−40 are how many?	50−10 are how many?
6. 60−30 are how many?	70−60 are how many?
7. 80−50 are how many?	90−70 are how many?

If the larger of the two numbers consists of tens and units both, while the smaller consists of tens only, their difference is found *by subtracting the tens of the smaller number from the tens of the larger number, and writing the units of the larger number on the right of the difference.*

8. A man having 36 sheep, sold 20 of them. How many had he left?

EXPLANATION. 36=3 tens and 6 units, and 20 units=2 tens, and 2 tens from 3 tens=1 ten, and 1 ten and 6 units =16, *Ans.*

9. 46−20 are how many?	19−10 are how many?
10. 39−10 are how many?	71−60 are how many?
11. 54−30 are how many?	83−30 are how many?
12. 63−20 are how many?	57−20 are how many?
13. 76−50 are how many?	87−50 are how many?
14. 85−60 are how many?	49−10 are how many?
15. 67−40 are how many?	66−50 are how many?

When both numbers consist of *units* and *tens, take units from units and tens from tens.* If, however, the unit figure of the smaller number is greater than that of the larger number, *take that unit figure from* 10, *and add the difference to the unit figure of the larger number; then diminish the* 10s *of the larger number by* 1 *more than the* 10s *of the smaller number.*

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50. What is the direction for subtracting tens? What is the rule for subtracting tens only from tens and units? What is the rule when *both numbers* consist of units and tens?

16. From 43 take 17.

EXPLANATION. 7 units cannot be taken from 3 units; therefore, take 7 from 10, and to the 3 left, add the 3 in the larger number, 43. This gives 6 as the unit figure of the answer. Then increase the 1 ten of the smaller number, 17, by 1, making it 2, and take the 2 from the 4 tens of the larger number. The difference is 2 tens. Then, on the right of this, place the 6 units, and the true difference is obtained, viz. 26.

17. From 51 take 23.
18. From 57 take 19.
19. From 83 take 55.
20. From 67 take 25.
21. From 83 take 52.
22. From 71 take 56.
23. From 85 take 38.
24. From 29 take 14.
25. From 93 take 58.
26. From 87 take 67.
27. From 73 take 68.
28. From 69 take 33.
29. From 88 take 79.
30. From 73 take 54.
31. From 29 take 13.
32. From 56 take 47.

SIMPLE SUBTRACTION, *written*.

§ **51.**—1. It is obvious from the definition of subtraction, (§ **46**,) that only two numbers can be employed in the operation, one of which is to be taken from, or out of, the other. The one to be taken from the other is called the *subtrahend*, and that from which the subtrahend is taken is called the *minuend*, while the number obtained is called *the remainder*.

2. Figures can be subtracted one from another *when they represent numbers merely*. If, however, they are employed to express definite kinds of objects, *they can be subtracted only when both represent the same kind of objects;* that is, bushels can be taken from bushels, dollars from dollars, &c.; but bushels cannot be taken from dollars.

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51. What is subtraction? (§ 46.) How many numbers employed? What is the one called? What, the other? What the number obtained? When can figures be subtracted? When, if they express definite objects?

RULE FOR SUBTRACTION.

Write down the two numbers, the smaller under the larger, placing units under units, and tens under tens, and take each lower figure from that standing directly above it. Whenever the lower figure is larger than that above it, take that lower figure from 10, *and to the difference add the upper figure, remembering always, whenever an upper figure is increased by* 10, *that the next lower figure must be increased by* 1, *before it is taken from the figure above it.*

After the sum is performed, the accuracy of the operation is tested by adding the remainder to the subtrahend; when, if the work has been accurately performed, *the sum of the two will be the minuend.*

1. From 8567 take 3425.

PERFORMED.

Minuend	8 5 6 7 larger number.
Subtrahend	3 4 2 5 smaller number.
Remainder	5 1 4 2

For this example no explanation is needed, each figure of the subtrahend being taken directly from the figure of the minuend standing above it.

2. From 7328 take 4167.

PERFORMED.

	7 3 2 8=min.
	4 1 6 7=sub.
	3 1 6 1=rem.

EXPLANATION. Taking 7 units from 8 units, 1 unit will remain, which is written down at the foot of the units' column. But 6 tens cannot be taken from 2 tens; we therefore take it from 10 tens, and to the remainder, 4 tens, we add the 2 tens of the minuend, and thus obtain 6 tens to be placed at the foot of the tens' column. To compensate for the 10 we assumed in subtracting the tens, we add 1 to the 1 hundred of the subtrahend, thus making it 2, and subtract the 2 from the 3 hundred of the minuend, and ob-

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Repeat the rule. How is the accuracy of the work tested?

tain a remainder of 1 hundred for the remainder. Lastly, we subtract the 4 thousand of the subtrahend from the 7 thousand of the minuend, and obtain a remainder of 3. We thus obtain the total remainder of 3161.

NOTE. The assuming of 10 from which to subtract the lower figure when larger than the figure standing above it, is called *borrowing* 10.

3. From 63814 subtract 45263,

PERFORMED.

6 3 8 1 4 = min.
4 5 2 6 3 = sub.
1 8 5 5 1 = rem.

I here say, 4−3=1, and 10−6=4, and 4+1=5; then 2+1=3, and 8−3=5; also 10−5=5, and 5+3=8, and finally, 4+1=5, and 6−5=1.

4. From 875643 subtract 395456. *Ans.* 480187.
5. From 396781 subtract 287884. *Ans.* 108947.
6. From 987654 take 456789. *Ans.* 530865.
7. From 876543 take 345678. *Ans.* 530865.
8. From 687957 take 397668. *Ans.* 290289.
9. From 596317 take 384567. *Ans.* 211750.
10. From 1073563 take 891563. *Ans.* 182000.
11. From 1364578 take 954631. *Ans.* 409947.
12. From 3895716 take 1934564. *Ans.* 1961152.
13. From 8643769 take 4897654. *Ans.* 3746115.
14. From 7183456 take 1745694. *Ans.* 5437762.
15. From 7788996 take 6939154. *Ans.* 849842.
16. From 3344556 take 2617478. *Ans.* 727078.
17. From 5566778 take 1956437. *Ans.* 3610341.
18. From 8811772 take 3456789. *Ans.* 5354983.
19. From 9076432 take 7819567. *Ans.* 1256865.
20. From 2345473 take 876921. *Ans.* 1468552.
21. From 8136741 take 5687345. *Ans.* 2449396.
22. From 7891234 take 1234789. *Ans.* 6656445.
23. From 8315794 take 7186432. *Ans.* 1129362.
24. From 9108765 take 3890787. *Ans.* 5217978.
25. From 3076059 take 1849718. *Ans.* 1226341.
26. From 7867564 take 2948675. *Ans.* 4918889.

What is stated in the note?

27. From 4716359 take 1975645. *Ans.* 2740714.
28. From 9182736 take 1928378. *Ans.* 7254358.
29. From 7891367 take 4986745. *Ans.* 2904622.
30. From 8867547 take 3956431. *Ans.* 4911116.
31. From 8715643 take 7146894. *Ans.* 1568749.
32. From 9182735 take 8197673. *Ans.* 985062.
33. From 1678909 take 871964. *Ans.* 806945.
34. From 6813957 take 2468035. *Ans.* 4345922.
35. From 8795643 take 6987869. *Ans.* 1807774.

§ **52.** It is often desirable to have some test by which to try the accuracy of an operation. The more convenient test for subtraction is, *to add the remainder, or difference between the two numbers, to the smaller number or subtrahend; their sums must equal the larger number or minuend. This is done on the obvious principle, that the difference between two numbers added to the smaller one must produce the greater.* The proof of the following sums is required.

1. From 875956 take 387618.

OPERATION.

$$\text{added} = \left\{\begin{array}{l} 875956 = \text{minuend.} \\ \underline{387618} = \text{subtrahend.} \\ 488338 = \text{remainder.} \end{array}\right.$$

$$\overline{875956} = \text{sub.} + \text{rem. and same as minuend.}$$

2. From 776356 take 631849. *Ans.* 144507.
3. From 8971234 take 3897612. *Ans.* 5073622.
4. From 3876956 take 1973468. *Ans.* 1902488.
5. From 6071945 take 2809876. *Ans.* 3262069.
6. From 7389145 take 5894387. *Ans.* 1494758.
7. From 6807901 take 4870309. *Ans.* 1937592.
8. From 9911234 take 1896475. *Ans.* 8014759.
9. From 5318796 take 2987654. *Ans.* 2341142.
10. From 8753104 take 6975367. *Ans.* 1777737.
11. From 6734689 take 3891968. *Ans.* 2842721.

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52. What is a convenient mode of proving subtraction? What must the sum of the remainder and subtrahend equal? On what principle is this done?

12. From 3896712 take 1838976. *Ans.* 2057736.
13. From 3781895 take 1967846. *Ans.* 1814049.
14. From 9123456 take 6543219. *Ans.* 2580237.

§ **53.** It often becomes necessary to subtract one number from the sum of two, three, or more numbers. To do this, first *add together the several numbers, and make their sum a minuend*, from which *subtract the given number.*

1. A man who had $350, received from his neighbor $1869. He afterward paid away $847. How much had he left? *Ans.* $1372.

It is obvious that $350+$1869=$2219 is the amount received; and that $2219−$847=$1372.

2. A man bought two farms adjoining each other, one of which contained 793 acres, and the other 968. From the two he sold 1680 acres. How many acres did he then have left? *Ans.* 81 acres.

3. The following three sums of money were paid a merchant, viz.: $793, $1196, and $1749, of which he paid away $1999. How much had he left? *Ans.* $1739.

4. A farmer raised 173 bushels of rye, 213 bushels of wheat, 136 bushels of corn, and 93 bushels of barley. Of these several kinds of grain he sold 217 bushels. How many bushels had he left? *Ans.* 398 bushels.

5. A man bought a farm for $3756, he then built a house and two barns which cost him as follows: the house, $1729, one barn, $1296, and the other $738. He then sold the farm and buildings for $9287. How much did he gain? *Ans.* $1768.

6. A man went a journey of two days, traveling by steamboat 167 miles, and by railroad 75 miles. On the first day of his return he traveled 187 miles by steamboat and railroad, how many miles then remained for his second day's journey? *Ans.* 55 miles

7. A collector received on Monday, $1632; on Tuesday. $963; on Wednesday, $1897, and on Thursday, $1397.

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53. What is the process of subtracting one number from the sum of two or more numbers?

He then paid over $3674. How much was left in his hands? *Ans.* $2215.

§ 54. It is frequently necessary to subtract the sum of two or more numbers from a single number. This is done *by adding the two or more numbers into one, and making their sum a subtrahend, while the single number forms the minuend.*

1. A drover bought of A. a flock of sheep for $729; of B. a drove of horned cattle for $2278; and a second flock of sheep for $588. He drove them all to market, and there received for them $4689. How much did he gain?

Ans. $1094.

The sum of $729+$2278+$588=$3595 must be subtracted from $4689 to obtain the true answer.

2. A. received from B. $8756; from C. $5749; from D. $2986; and from E. $3371. He then bought a farm for $12856. How much did he have left? *Ans.* $8006.

3. Farmer A. raised 715 bushels of oats, 963 bushels of rye, 569 bushels of wheat, and 357 bushels of corn; while farmer B. raised in all, 3179 bushels of grain. How many bushels did B. raise more than A.? *Ans.* 575 bushels.

4. Having in my possession $1396; I paid A. $473, B. $321, C. $129, and D. $113. How many dollars had I left? *Ans.* $360.

5. A butcher dressed an ox which weighed 1473 pounds. His tallow weighed 83 pounds; his hide, 68 pounds; one of his fore-quarters, 298 pounds; the other, 260 pounds; and one of his hind-quarters, 360 pounds. Required the weight of the other hind-quarter. *Ans.* 404 pounds.

6. A man having a journey of 1567 miles to perform, traveled Monday, 341 miles; Tuesday, 299 miles; and on Wednesday, 462. How far had he then to travel?

Ans. 465 miles.

7. A man having $7896, bought a farm for $3280, on which he built a barn which cost him $796. He also bought

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54. How is the sum of two or more numbers taken from a single number?

a horse for $115, a yoke of oxen for $93, and 5 cows for $158. How much money had he left? *Ans.* $3454.

8. A farmer raised 796 bushels of potatoes. Of these he sold 286 bushels, and fed out to his cattle 321 bushels. How many bushels had he left? *Ans.* 189 bushels.

9. A man, whose property is valued at $9973, owes A. $1689, B. $2109, and C. $3087. How many dollars were left him, after his debts were paid? *Ans.* $3088.

§ **55.** Whenever there are several numbers given, whose amount is to be taken from the amount of several other given numbers, *the former numbers must be added together for a subtrahend, while the sum of the latter must form a minuend;* their *difference* will then be the number sought.

1. A man received from A. $727; from B. $963; and from C. $842. He then paid D. $642; E. $753; and F. $599. How much had he left?

PERFORMED.

727	642	then 2532 = cash received.
963	753	1994 = " paid out.
842	599	$538 = " left in hand.
$2532 = money received.	$1994 = money paid out.	

2. A farmer has three flocks of sheep, numbering as follows: The first, 726; the second, 1137; and the third, 921. How many will he have left, after selling A. 642, B. 379, and C. 596? *Ans.* 1167.

3. If a farmer raise 376 bushels of oats, 576 bushels of corn, 631 bushels of rye, and 227 of wheat, and sell 193 bushels of oats, 287 bushels of corn, 320 bushels of rye, and 188 bushels of wheat, how many bushels of grain will he have left? *Ans.* 822 bushels.

4. There was exported from the United States in the year 1847, dried fish to the amount of $659629; pickled fish, $136221; and of whale and other fish oils, $1070659.

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55. How proceed when the amount of several numbers is to be taken from the amount of several other numbers?

In 1848, the value of the exports of the same articles was, of dried fish, $609482; of pickled fish, $109315; and of whale and other fish oils, $552388. How much did the value of the exports of these articles for the year 1847 exceed those of the year 1848? *Ans.* $595324.

5. Of the products of the forest there were exported in the year 1847, skins and furs to the amount of $747145; hewn and sawn timber, $1849911; and of other lumber, $342781. For the year 1848, the exports of the same were valued as follows: furs and skins, $607780; hewn and sawn timber, $2429863; and of other lumber, $283433. How much did the exports of these articles for 1848 exceed those of 1847 in value? *Ans.* $381239.

6. In the year 1847, the following articles were exported, in value as follows: viz., masts and spars, $23270; oak and other barks, $95355; manufactures of wood, $1495924. The following were the exports of the same for 1848: viz., masts and spars, $129760; oak and other barks, $184126, and of wooden manufactures, $2042695. How much does the value of these exports for 1848 exceed that of the same articles for 1847? *Ans.* $742032.

7. The following animal products were exported in the year 1847, and valued as follows: beef, tallow, hides, and horned cattle, $2434003; butter and cheese, $1741770; pork, bacon, lard, and live hogs, $6630842, In the year 1848, the value of the same was as follows: beef, tallow, hides, and horned cattle, $1905341; butter and cheese, $1361668; pork, bacon, lard, and live hogs, $9003272, How much does the value of these exports for 1848 exceed that of the same articles for 1847? *Ans.* $1463666.

8. The following vegetable products were exported in the year 1847, in value as follows: wheat, $6049350; flour, $26133811; and Indian corn, $14395212. In the year 1848 the same articles were exported, in value as follows: viz., wheat, $2669175; flour, $13194109; and Indian corn, $3837483. How much does the value of the exports of 1847 exceed those of 1848?

Ans. $26877606.

9. The following were the exports of other vegetable products for the same years: viz., for the year 1847, Indian

meal was exported to the value of $4301334; rye meal, $225502; rye, oats, and other small grain, $1600962. For the year 1848, Indian meal, $1807601; rye meal, $174566; rye, oats, and other small grain, $376572. How much do the products of 1847 exceed in value those of 1848?

Ans. $3769059.

10. The exports of potatoes in 1847 were valued at $109062; of apples, $92961; rice, $3605896: in 1848, of potatoes, $86277; apples, $88944; rice, $2331824. How much did their value for 1847 exceed that of 1848?

Ans. $1300874.

11. In 1847, the value of tobacco exported was $7242086, and that of cotton $53415848; while for 1848, the value of exported tobacco was $7551122, and of cotton $61998294. How much do the exports of 1848 exceed those of 1847?

Ans. $8891482.

12. During the year 1847, the value of flax-seed exported was $1346; of hops, $150654; brown sugar, $25483; indigo, $10. During the year 1848, the value of the same articles exported was, flax-seed, $1584; hops, $17671; brown sugar, $8891; indigo, $1100. How much did the exports of 1848 exceed those of 1847? *Ans.* $148247.

13. The following articles of manufacture were exported during the year 1847, to the following amounts: viz., soap and tallow candles, $606798; leather boots and shoes, $243816; household furniture, $225700. In 1848, the value of the same articles exported was, of soap and tallow candles, $670223; leather boots and shoes, $194095; household furniture, $297358. How much do the exports of 1848 exceed those of 1847? *Ans.* $85362.

14. During the year 1847, carriages of different kinds were exported from the United States to the value of $75369; hats to the amount of $59536; and saddlery to the amount of $13102. For the year 1848, the exports of these articles were, of carriages, $89963; of hats, $55493; and of saddlery, $27435. How much did the exports of 1848 exceed those of 1847? *Ans.* $24884.

15. Of the following articles during the year 1847, the exports were valued as follows: viz., of wax, $161527; *spirits distilled* from grain, $67781; of beer, ale, porter,

and cider, $68114. Those of the year 1848 were valued as follows: of wax, $134577; of spirits distilled from grain, $90957; of beer, ale, porter, and cider, $78071. What is the value of the exports for the year 1848 over those of 1847? *Ans.* $6183.

16. The value of snuff and tobacco exported during the year 1847, was $658950; linseed oil and turpentine, $498110; cordage, $27054. The value of the same articles for 1848 was, snuff and tobacco, $568435; linseed oil and turpentine, $31404; and of cordage, $29911. How much did the value of these exports of 1847 exceed those of 1848? *Ans.* $554364.

17. Also of pig and bar iron, and nails for 1847, the value of exports was, $168817; of iron castings, $69889; and of manufactures of iron, $929778. Of the same articles for 1848, the exports were, of pig and bar iron, and nails, $154036; of iron castings, $83188; and of manufactures of iron, $1022408. How much did the value of these exports for 1848 exceed those of 1847? *Ans.* $91,148.

18. There was received from the Croton water-works, from Oct. 5th, 1842, to May 1st, 1843, $17838; from May 1st, 1843, to May 1st, 1844, $91790; from May 1st, 1844, to May 1st, 1845, $118582; from May 1st, 1845, to May 1st, 1846, $164532; from May 1st, 1846, to May 1st, 1847, $194551; from May 1st, 1847, to May 1st, 1848, $226551; and from May 1st, 1848, to June 31st, 1849, $234286. There was expended on the same during the same years, the following sums, from year to year, viz.: $233198; $75411; $58433; $53403; $71565; and $67062. How much were the receipts above the expenditures?

Ans. $489058.

19. The coinage of gold at the United States mint for the year 1845, was $3756447; of silver, $1873200; and of copper, $38948. That of 1846 was, of gold, $4034177; of silver, $2558580; and of copper, $41208. How much does the coinage of 1846 exceed that of 1845?

Ans. $965370.

Coinage means the act of coining or making money of gold, silver, and copper.

20. The coinage of 1847 was, of gold, $20221385; of

silver, $2374450; and of copper, $61836. That of 1848 was, of gold, $3775512; of silver, $2640050; and of copper, $46158. How much does the coinage of 1847 exceed that of 1848? *Ans.* $16195951.

21. The receipts into the treasury of the United States for the first quarter of the year 1847, were $6153826; for the second quarter, $3641192; for the third quarter, $6319041; and for the fourth quarter, $7633804. During the year 1848 the receipts were as follows: first quarter, $11106257; second quarter, $5379152; third quarter, $9383092; fourth quarter, $5888567. How much do the receipts of the year 1848 exceed those of 1847?

Ans. $8009205.

22. There was paid from the treasury of the United States during the year 1847, for the support of light-houses, &c., the sum of $501018; for the survey of public lands, $144013; and for support of marine hospitals, $123257. During the year 1848, there was paid for the same objects the following sums, viz., $419277, $169902, and $140995. How much was expended for these objects in 1847 more than in 1848? *Ans.* $38114.

23. There was also paid from the treasury of the United States during the year 1847, the following sums, viz.: for building custom-houses, $54062; for surveying the coasts of the United States, $111000; for the mint establishment, $89972; also for the same purposes during the year 1848 the following sums, viz.: for custom-houses, $92140; for survey of coasts, $146000; and for the mint, $75850. How much was paid out for these purposes in 1848 more than in 1847? *Ans.* $58956.

24. The expenses for the support of the army of the United States for the year 1847, was $17880842; for the military academy, $124339; for fortifications, &c., $932962; and for armories, arsenals, &c., $1617216. For the same objects there was expended in 1848, the following sums, viz.: for the army, $18939155; for military academy, $130537; for fortifications, &c., $313743, and for armories, arsenals, &c., $1306486. How much did the expenses of 1848 exceed those of 1847? *Ans.* $134562.

§ 56. General Application of the Principles involved in the foregoing sections.

1. The annual transportation of the mail in the state of Maine, amounts to 1211635 miles, while in New Hampshire it amounts to only 836352 miles. How much does that of Maine exceed that of New Hampshire?

Ans. 375283 miles.

2. The annual cost of transporting the mail in Maine is $42565, while that of New Hampshire is $26242. How much more does it cost in Maine than in New Hampshire?

Ans. $16323.

3. In Vermont the annual transportation of the mail amounts to 757246 miles, while in Massachusetts it amounts to 1989630 miles. How much does that of Massachusetts exceed that of Vermont yearly?

Ans. 1232384 miles.

4. The salary of the president of the United States is $25000, while that of the secretary of state is $6000; the secretary of the treasury, $6000; the secretary of war, $6000; the secretary of the navy, $6000; the secretary of the interior, $6000; the postmaster-general, $6000; the attorney-general, $4000. How much do the salaries of all the secretaries, including that of the postmaster-general and the attorney-general, exceed that of the president of the United States?

Ans. $15000.

5. The expenses for the department of state, for the year ending June 30, 1850, were $64600, while those of the treasury department were $475258. How much do the expenses of the treasury department exceed those of the department of state? *Ans.* $411658.

6. In North Carolina the annual transportation of the mail amounts to 1623544 miles; while that of South Carolina amounts to 1058500. How many miles of transportation annually in North Carolina more than in South Carolina? *Ans.* 565044 miles.

7. The total annual expenditures of the post-office department for the years 1846, 1847, and 1848, were $4084297, *$3978570,* and $4326850; while the total receipts for the

same years were $3487199, $3955893, and $4371077. How much did the expenditures exceed the receipts?

Ans. $575548.

8. The receipts from postage on letters, papers, and pamphlets for the year 1841, was $4499528; while that of 1842, was $4546849. How much do those of 1842 exceed those of 1841? *Ans.* $47321.

9. White cotton goods were exported from the United States during the year 1847 to the amount of $3345902; printed cotton goods, $281320; and nankeen goods, $8794. The exports of the same for 1848 were, white cotton, $4866559; printed cotton, $351169, and nankeen, $2365. How much do the receipts from these articles for 1848 exceed those of 1847? *Ans.* $1584077.

10. During the year 1848, the tonnage of American vessels that entered the British American colonies was 867240, while that which cleared was 859791. What was the tonnage entered above what cleared? *Ans.* 7449 tons.

11. During the year 1848, the total value of exports from Maine was $1957395; from New Hampshire, $8243; from Vermont, $534102; from Massachusetts, $13419699; from Rhode Island, $221631; from Connecticut, $501061; while at the same time from New York alone the value of exports was $53351157. How far do the exports from New York alone exceed those from all the other states named? *Ans.* $36709026.

12. The total value of imports into the several New England states for the year 1848, was as follows: into Maine, $795565; New Hampshire, $61303; Vermont, $306005; Massachusetts, $28647707; Rhode Island, $351590; Connecticut, $229310; while into New York alone it amounted to $94525141. How much do the imports into New York exceed those of all the New England states? *Ans.* $64133661.

13. How much did the value of the imports into the Middle states during the year 1848 excel that of the exports from the same states; the imports to New York being

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Which are the New England states? Which are the Middle states?

$94525141; to New Jersey, $1835; to Pennsylvania, $12157585; and to Delaware, $490; while the exports from New York amounted to $53351157; from New Jersey, to $62; from Pennsylvania, to $5732333; and from Delaware, to $83058? *Ans.* $47518441.

14. During the year 1848, public lands were sold to the amount of $2030668; the year 1847, to $3296404; the year 1846, to $2904637; and during the year 1845, to $2470303. How much does the amount received during the years 1845 and 1846, excel that received during the years 1847 and 1848? *Ans.* $47868.

15. A gentleman possessed of an estate valued at $193756, made his will dividing it among his 4 sons. To the eldest he gave $57846; to the second, $51376; to the third, $49576; and the remainder to the fourth. How much did the fourth son receive? *Ans.* $34958.

16. A gentleman has real estate valued at $193756, and personal property to the amount of $58909. He at the same time owes A. $597; B. $6873; C. $12197; and D. $13131. How much will he have left after paying all his debts? *Ans.* $219867.

17. Suppose a man buy three farms valued as follows: one at $5648; another, at $7756; and a third, at $9372: and he afterward sell the same farms together for $30693. How much does he gain by the operation? *Ans.* $7917.

MULTIPLICATION.

§ 57. Multiplication consists in repeating one number as many times as there are units in another, thereby producing a number which contains either of the given numbers as many times as there are units in the other.

The following table must first be made familiar.

In what does multiplication consist?

MULTIPLICATION TABLE.

1× 1= 1	2× 1= 2	3× 1= 3	4× 1= 4
1× 2= 2	2× 2= 4	3× 2= 6	4× 2= 8
1× 3= 3	2× 3= 6	3× 3= 9	4× 3= 12
1× 4= 4	2× 4= 8	3× 4= 12	4× 4= 16
1× 5= 5	2× 5= 10	3× 5= 15	4× 5= 20
1× 6= 6	2× 6= 12	3× 6= 18	4× 6= 24
1× 7= 7	2× 7= 14	3× 7= 21	4× 7= 28
1× 8= 8	2× 8= 16	3× 8= 24	4× 8= 32
1× 9= 9	2× 9= 18	3× 9= 27	4× 9= 36
1×10= 10	2×10= 20	3×10= 30	4×10= 40
1×11= 11	2×11= 22	3×11= 33	4×11= 44
1×12= 12	2×12= 24	3×12= 36	4×12= 48

5× 1= 5	6× 1= 6	7× 1= 7	8× 1= 8
5× 2= 10	6× 2= 12	7× 2= 14	8× 2= 16
5× 3= 15	6× 3= 18	7× 3= 21	8× 3= 24
5× 4= 20	6× 4= 24	7× 4= 28	8× 4= 32
5× 5= 25	6× 5= 30	7× 5= 35	8× 5= 40
5× 6= 30	6× 6= 36	7× 6= 42	8× 6= 48
5× 7= 35	6× 7= 42	7× 7= 49	8× 7= 56
5× 8= 40	6× 8= 48	7× 8= 56	8× 8= 64
5× 9= 45	6× 9= 54	7× 9= 63	8× 9= 72
5×10= 50	6×10= 60	7×10= 70	8×10= 80
5×11= 55	6×11= 66	7×11= 77	8×11= 88
5×12= 60	6×12= 72	7×12= 84	8×12= 96

9× 1= 9	10× 1= 10	11× 1= 11	12× 1= 12
9× 2= 18	10× 2= 20	11× 2= 22	12× 2= 24
9× 3= 27	10× 3= 30	11× 3= 33	12× 3= 36
9× 4= 36	10× 4= 40	11× 4= 44	12× 4= 48
9× 5= 45	10× 5= 50	11× 5= 55	12× 5= 60
9× 6= 54	10× 6= 60	11× 6= 66	12× 6= 72
9× 7= 63	10× 7= 70	11× 7= 77	12× 7= 84
9× 8= 72	10× 8= 80	11× 8= 88	12× 8= 96
9× 9= 81	10× 9= 90	11× 9= 99	12× 9=108
9×10= 90	10×10=100	11×10=110	12×10=120
9×11= 99	10×11=110	11×11=121	12×11=132
9×12=108	10×12=120	11×12=132	12×12=144

MENTAL EXERCISES IN MULTIPLICATION.

1	times	1	are	how many?	2	times	1	are	how many?
1	"	3	"	"	2	"	3	"	"
1	"	2	"	"	2	"	2	"	"
1	"	5	"	"	2	"	5	"	"
1	"	4	"	"	2	"	4	"	"
1	"	7	"	"	2	"	7	"	"
1	"	6	"	"	2	"	6	"	"
1	"	9	"	"	2	"	9	"	"
1	"	8	"	"	2	"	8	"	"
1	"	10	"	"	2	"	10	"	"
1	"	12	"	"	2	"	12	"	"
1	"	11	"	"	2	"	11	"	"

3	times	1	are	how many?	4	times	1	are	how many?
3	"	3	"	"	4	"	3	"	"
3	"	2	"	"	4	"	2	"	"
3	"	5	"	"	4	"	5	"	"
3	"	4	"	"	4	"	4	"	"
3	"	7	"	"	4	"	7	"	"
3	"	6	"	"	4	"	6	"	"
3	"	9	"	"	4	"	9	"	"
3	"	8	"	"	4	"	8	"	"
3	"	10	"	"	4	"	10	"	"
3	"	12	"	"	4	"	12	"	"
3	"	11	"	"	4	"	11	"	"

5	times	1	are	how many?	6	times	1	are	how many?
5	"	3	"	"	6	"	3	"	"
5	"	2	"	"	6	"	2	"	"
5	"	5	"	"	6	"	5	"	"
5	"	4	"	"	6	"	4	"	"
5	"	7	"	"	6	"	7	"	"
5	"	6	"	"	6	"	6	"	"
5	"	9	"	"	6	"	9	"	"
5	"	8	"	"	6	"	8	"	"
5	"	10	"	"	6	"	10	"	"
5	"	12	"	"	6	"	12	"	"
5	"	11	"	"	6	"	11	"	"

7	times	1	are	how many ?	8	times	1	are	how many ?
7	"	3	"	"	8	"	3	"	"
7	"	2	"	"	8	"	2	"	"
7	"	4	"	"	8	"	4	"	"
7	"	6	"	"	8	"	6	"	"
7	"	5	"	"	8	"	5	"	"
7	"	7	"	"	8	"	7	"	"
7	"	9	"	"	8	"	9	"	"
7	"	8	"	"	8	"	8	"	"
7	"	10	"	"	8	"	10	"	"
7	"	12	"	"	8	"	12	"	"
7	"	11	"	"	8	"	11	"	"

9	times	1	are	how many ?	10	times	1	are	how many ?
9	"	3	"	"	10	"	3	"	"
9	"	2	"	"	10	"	2	"	"
9	"	4	"	"	10	"	4	"	"
9	"	6	"	"	10	"	6	"	"
9	"	5	"	"	10	"	5	"	"
9	"	7	"	"	10	"	7	"	"
9	"	9	"	"	10	"	9	"	"
9	"	8	"	"	10	"	8	"	"
9	"	10	"	"	10	"	10	"	"
9	"	12	"	"	10	"	12	"	"
9	"	11	"	"	10	"	11	"	"
9	"	13	"	"	10	"	13	"	"

11	times	1	are	how many ?	12	times	1	are	how many ?
11	"	3	"	"	12	"	3	"	"
11	"	2	"	"	12	"	2	"	"
11	"	4	"	"	12	"	4	"	"
11	"	6	"	"	12	"	6	"	"
11	"	5	"	"	12	"	5	"	"
11	"	7	"	"	12	"	7	"	"
11	"	9	"	"	12	"	9	"	"
11	"	8	"	"	12	"	8	"	"
11	"	10	"	"	12	"	10	"	"
11	"	12	"	"	12	"	12	"	"
11	"	11	"	"	12	"	11	"	"
11	"	13	"	"	12	"	13	"	"

§ 58.—1. James bought 3 oranges at 2 cents apiece. How many cents did they cost?

2. George bought 6 lemons at 3 cents each. How many cents did they all cost?

3. What will 3 yards of cloth cost at $3 a yard? 5 yards at $4 per yard?

4. What cost 7 barrels of flour at $5 a barrel? 6 barrels at $7 a barrel?

5. What cost 8 hundredweight of sugar at $8 a hundredweight? 9 hundredweight at $6 a hundredweight?

6. What cost 7 reams of paper at $3 a ream? at $5 a ream, what cost 8 reams?

7. What cost 9 yards of cloth at $7 a yard? 11 yards at $5 a yard?

8. What cost 11 pounds of veal at 8 cents a pound? 9 pounds at 6 cents a pound?

9. In 1 penny there are 4 farthings. How many farthings in 7 pence? in 10 pence? in 6 pence?

10. In 9 pence how many farthings? in 5 pence? in 7 pence? in 11 pence? in 12 pence?

11. In 1 shilling there are 12 pence. How many pence in 5 shillings? in 7 shillings? in 9 shillings?

12. How many pence in 8 shillings? in 9 shillings? in 11 shillings? in 10 shillings? in 12 shillings?

13. How many pence in 3 shillings? in 5 shillings? in 8 shillings? in 6 shillings?

14. In 1 gallon there are 4 quarts. How many quarts in 5 gallons? 7 gallons?

15. How many quarts in 11 gallons? in 10 gallons? in 12 gallons? in 7 gallons?

16. In 1 quart there are 8 gills. How many gills in 5 quarts? in 9 quarts?

17. How many gills in 7 quarts? in 3 quarts? in 8 quarts? in 11 quarts?

18. In 1 pint there are 4 gills. How many gills in 9 pints? in 11 pints? in 7 pints?

19. How many gills in 8 pints? in 6 pints? in 4 pints? in 12 pints? in 5 pints?

20. What cost 8 barrels of molasses at $9 a barrel? at *$8 a barrel? at $11* a barrel?

21. What cost 12 barrels of flour at $7 a barrel? at $9 a barrel? at $10 a barrel?

22. If a stage run 9 miles in 1 hour, how far will it run in 7 hours? in 12 hours?

23. How many trees are there in an orchard of 12 rows of trees, each row containing 11 trees?

24. If two men travel in opposite directions, the one 2 miles an hour, and the other 3 miles an hour, how far will they be apart in 6 hours?

EXPLANATION. They travel in opposite directions, the one 2 and the other 3 miles in 1 hour. They will then be 5 miles apart in 1 hour, and 6 times 5 miles=30 miles, in 6 hours.

25. Two vessels sail from the same harbor, but in opposite directions, the one 3 and the other 4 miles per hour. How far are they apart at the end of 9 hours? 7 hours? 5 hours? 11 hours? 12 hours? 8 hours?

26. Suppose the two vessels sail the same way, the one at the rate of 11 miles an hour, the other at the rate of 7 miles per hour. How far will they be apart at the end of 10 hours? 9 hours? 8 hours? 12 hours? 11 hours?

27. If it require 9 men to build a wall in 11 days, how many men will build it in 1 day? how many in 1 day if 9 build it in 9 days?

28. How many men must be employed to perform in 1 day as much labor as 11 men can perform in 12 days? in 9 days? in 7 days? in 10 days?

29. If 12 men do a piece of work in 7 days, how many days' work do they perform.

30. If a man earn $12 in 1 month, how much will he earn in 11 months? in 7 months? in 9 months? in 8 months? in 6 months?

§ **59.** When either of the numbers is composed of a number not greater than 12 with a cipher or ciphers on its right hand, they may be multiplied *mentally, by omitting the cipher or ciphers and multiplying the other figures as*

59. How multiply when there are ciphers on the right of either of the given numbers?

before, and finally placing on the right of the product as many ciphers as were omitted.

1. A farmer sold 60 sheep for 7 dollars each. How much did he receive for them?

OPERATION. 60 (cipher omitted)=6, and $6 \times 7 = 42$, and 42 with the cipher placed on its right is $420. The answer is therefore $420.

2. A farmer sold 9 horses for $90 each. How much did he receive for them all?

3. How far will a man travel in 11 days, if he travel 40 miles a day? if he travel 60 miles a day? 30 miles a day? 50 miles? 70 miles? 80 miles?

4. How far will a man travel in 12 days, if he travel 20 miles a day? 30 miles? 40 miles? 50 miles? 60 miles? 70 miles? 80 miles? 90 miles? 100 miles?

5. If a cow cost 30 dollars, how much will 9 cows cost? 2 cows? 11 cows? 8 cows? 7 cows? 12 cows? 3 cows? 5 cows? 6 cows?

6. If a farmer raise 50 bushels of corn each year, how many bushels will he raise in 10 years? in 6 years? in 8 years? in 12 years? in 7 years?

7. If it cost $900 to build a house, how much will it cost to build 8 houses? 9 houses? 11 houses? 12 houses? 8 houses? 7 houses?

8. If a locomotive run 300 miles in 1 day, how far will it run in 7 days? in 8 days? 9 days? 11 days? 12 days?

9. If there are 40 lines on 1 page of a book, how many lines are there on 12 pages? 7 pages? 5 pages? 6 pages? 8 pages? 11 pages?

10. If a book contain 400 pages, and there is an average of 50 lines on a page, how many lines are there in the book?

Ans. 20000 lines.

§ **60.** If both the numbers have ciphers on their right, *reject the ciphers, and multiply the significant figures only; then place on the right of the product as many ciphers as were rejected from both numbers.*

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60. How multiply if there are ciphers on the right of both the *given numbers?*

1. What will 40 tons of hay cost at $20 a ton?

   4×2=8, and 8 with the two ciphers=$800.

2. What is the product of 30 multiplied by 60?
3. What is the product of 80 multiplied by 60?
4. What is the product of 50 multiplied by 70?
5. What is the product of 90 multiplied by 40?
6. What is the product of 700 multiplied by 20?
7. What is the product of 80 multiplied by 30?
8. What is the product of 50 multiplied by 30?
9. What is the product of 20 multiplied by 90?
10. What is the product of 70 multiplied by 30?
11. What is the product of 80 multiplied by 70?

§ 61. If one of the numbers is composed of units and tens, they can be multiplied mentally, *by multiplying the units and tens separately and then adding the products together.*

1. A farmer bought 23 sheep at 3 dollars each, how much did he pay for the whole number?

OPERATION. 23=20+3, or 2 tens and 3 units. Then 20 sheep at $3=$60; and 3 sheep at $3=$9. Then $60+$9=$69, *Ans.*

2. What will 32 loads of hay cost at $9 a load? at $10 a load?
3. If one acre of land cost $37, what will 7 acres cost? 9 acres? 8 acres?
4. What will 7 rolls of broadcloth cost at $31 a roll? at $28? at $36?
5. What will 9 loads of wood cost at $17 a load? $15 a load? $12 a load?
6. In 1 gallon there are 4 quarts. How many quarts in 26 gallons? in 33 gallons?
7. In 1 pound there are 16 ounces. How many ounces in 8 pounds? in 11 pounds?
8. In 1 quarter of a hundredweight there are 25 pounds. How many pounds in six quarters of a hundredweight? in 7 quarters? in 9 quarters?
9. In 1 pistole there are 22 shillings. How many shillings in 6 pistoles? in 8 pistoles? in 4 pistoles? in 7 *pistoles?*

10. In 1 guinea there are 28 shillings. How many shillings in 4 guineas? in 7 guineas? in 5 guineas? in 8 guineas?

SIMPLE MULTIPLICATION, *written.*

§ **62.**—1. In multiplication *we repeat one number as many times as there are units in another.* (§ **57.**) Suppose it is required to find the amount of 4 times 17. This is determined by multiplication. Thus we say, 4 times 7 is 28, and write down the 8 only; we then say 4 times 1 are 4 and 2 to carry are 6; we then write the 6 on the left of the 8, and thus obtain the true product, 68. The same result is obtained by writing 17 down 4 times and adding; that is four 17s added together give the same result as one 17 multiplied by 4; for $17\times4=68$, and $17+17+17+17=68$.

2. Two numbers are always required for the performance of any operation in multiplication. These numbers are called, the one the *multiplicand*, the other the *multiplier*. The number which results from multiplying two numbers together is called the *product*. Thus in the operation above, 17 *is the multiplicand*, 4 *the multiplier, and* 68 *the product.*

3. The multiplier and multiplicand together are called *factors*. By this we mean that they are the makers or producers of the number obtained, that is the product.

4. The product is obtained by repeating the multiplicand; it must therefore agree with it in denomination.

5. The multiplier is to be considered merely as an abstract number, expressing simply the number of repetitions to which the multiplicand is subjected.

6. It is usual in multiplication to make the larger number the multiplicand, and the smaller one the multiplier.

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62. How often is one number repeated in multiplication? How else may the same result be obtained? How many numbers are employed? What called? What is the resulting number called? What are the multiplicand and multiplier together called? What do we *mean by that?* *How* is the product obtained? With what must it *agree?* *How* consider the multiplier? Expressing what?

The truth or accuracy of the result does not however depend on this order of multiplication; for $12 \times 4 = 4 \times 12$; that is we obtain the same product, viz. 48, whether we multiply 12 by 4, or 4 by 12. Convenience is however promoted by multiplying the larger by the smaller number.

7. The sign or character by which the multiplication of one number by another is indicated, consists of an oblique cross, ($\times$).

§ 63. To Multiply by any Number not greater than 12.

RULE.

Write the multiplier under the multiplicand; and, beginning with the right-hand figure of the multiplicand, multiply each figure by the multiplier, and place the number produced under the figure multiplied, in all cases whenever that number is less than 10. *If, however, the number produced be more than* 10, *write down only the right-hand figure of that number, and add the* 10*s to the product of the next figure of the multiplicand.*

(*a*) The entire product of the left-hand figure of the multiplicand must in all cases be written down.

(*b*) Whenever the multiplier consists of units only, *the denomination of each figure of the product will be the same as that of the figure multiplied.*

1. What is the product of 456 multiplied by 8?

Performed.
$$\begin{array}{r} 4\,5\,6 = \text{multiplicand.} \\ 8 = \text{multiplier.} \\ \hline 3\,6\,4\,8 = \text{product.} \end{array}$$

Explanation. I here say 8 times 6 are 48, and writing down the 8, add the 4 to the product of 5 multiplied by 8.

What is usual? Is this necessary? Illustrate. What is the sign of multiplication?

63. How write the numbers? Begin with what figure? How proceed? If the number produced be more than 10, how proceed? What must be done relative to the product of the left-hand figure? Whenever the multiplier consists of units, what is the denomination of each figure of the product?

That is, I say 8 times 5 are 40 and 4 added make 44, and writing down the right-hand 4, I add the left-hand one to the product of 4 multiplied by 8, and obtain 36; the whole of which is written down, it being the product of the left hand figure of the multiplicand.

2. Multiply 7246 by 7.

PERFORMED. 7 2 4 6=multiplicand.
7=multiplier.
5 0 7 2 2=product.

EXPLANATION. $6\times7=42$; write down 2 and carry 4. $4\times7=28$, and $28+4=32$; write down 2 and carry 3. And again, $7\times2=14$, and $14+3=17$; write down 7 and carry 1; and finally, $7\times7=49$, and $49+1=50$, all of which is written down, thus producing the entire product 50722.

3. Multiply 85637 by 5. *Ans.* 428185.
4. Multiply 77656 by 4. *Ans.* 310624.
5. Multiply 356789 by 6. *Ans.* 2140734.
6. Multiply 209378 by 9. *Ans.* 1884402.
7. Multiply 675436 by 10. *Ans.* 6754360.
8. Multiply 587637 by 11. *Ans.* 6464007.
9. Multiply 783547 by 12. *Ans.* 9402564.
10. Multiply 776633 by 7. *Ans.* 5436431.
11. What is the value of 376 yards of broadcloth at $7 per yard? *Ans.* $2632.
12. How much can a man earn by 378 days' labor, if he receive $3 per day? *Ans.* $1134.
13. What will 493 barrels of flour cost at $7 a barrel? *Ans.* $3451.
14. A hatter has in store 539 hats, of an average value of $4. What is the total value? *Ans.* $2156.
15. How much will 163 loads of hay come to, at $12 a load? *Ans.* $1956.
16. How far can a man travel in 276 hours if he travel 9 miles an hour? *Ans.* 2484 miles.
17. A man bought a farm containing 789 acres and paid $8 an acre. How much did the whole cost? *Ans.* $6312.
18. What will 3785 pounds of beef cost at 11 cents a *pound?* *Ans.* 41635 cents.

19. If a pound of sugar cost 7 cents, what will 8958 pounds cost? *Ans.* 62706 cents.

20. If the value of a load of straw be $6, what are 989 loads worth? *Ans.* $5934.

21. In 1 foot there are 12 inches. How many inches in 678 feet? *Ans.* 8136 in.

22. In 1 mile there are 8 furlongs. How many furlongs in 1272 miles? *Ans.* 10176 fur.

23. How many furlongs in 1369 miles. *Ans.* 10952 furlongs. How many in 667 miles? *Ans.* 5336 furlongs. How many in 2981 miles? *Ans.* 23848. In 17685 miles? *Ans.* 141480.

24. In 1 furlong there are 40 rods. How many rods in 976 furlongs? (§ 56.) *Ans.* 39040 rods.

25. How many rods are there in 897 furlongs? *Ans.* 35880 rods.

26. In 1 pound sterling there are 20 shillings. How many shillings in 886 pounds? *Ans.* 17720*s.*

27. How many shillings in 1789 pounds? *Ans.* 35780*s.* In 5683 pounds? *Ans.* 113660*s.* In 3587 pounds? *Ans.* 71740*s.* In 1859 pounds? *Ans.* 37180*s.*

28. In 1 shilling there are 12 pence. How many pence in 653 shillings? *Ans.* 7836 pence. In 859 shillings? *Ans.* 10308 pence. In 8764 shillings? *Ans.* 105168 pence. In 6853 shillings? *Ans.* 82236 pence.

29. In 653 pounds how many pence? (Multiply first by 20, and then by 12. See sums 26 and 28.) *Ans.* 156720 pence.

30. In 1875 pounds how many pence? *Ans.* 450000 pence. In 2163 pounds? *Ans.* 519120 pence.

31. In 1 penny there are four farthings. How many farthings in 1684 pence? *Ans.* 6736 farthings.

32. In 2856 pence how many farthings? *Ans.* 11424. How many in 2196 pence? *Ans.* 8784.

33. In 1728 pence how many farthings? *Ans.* 6912. In 3756 pence? *Ans.* 15024.

34. In 576 pounds how many farthings? (See sums 26, 28, and 31.) *Ans.* 552960 farthings.

35. In 358 pounds how many farthings? *Ans.* 343680. In 297 *pounds*? *Ans.* 285120.

36. In 569 pounds how many farthings? *Ans.* 546240. In 392 pounds? *Ans.* 376320.

37. If it cost 6 cents to ride 1 mile, how many cents will it cost to ride 896 miles? *Ans.* 5376 cents.

38. If 1 yard of cloth cost \$9, what will 2742 yards cost? *Ans.* \$24678.

39. What is the cost of 389 yards of cloth at 7 shillings per yard? *Ans.* 2723 shillings.

40. What will a farm of 348 acres cost at \$11 per acre? *Ans.* \$3828.

41. What will be the cost of 983 yards of broadcloth at 5 dollars per yard? *Ans.* \$4915.

§ **64.** To Multiply by any Number embraced in the Multiplication Table; that is, any Composite Number between 12 and 144.

1. A *composite number* is one produced by the multiplication of *two or more smaller numbers.* Such is the character of the numbers of the multiplication table, they being in each case the *product of two smaller numbers.* The smaller numbers are called the *factors* of the larger numbers, produced by their multiplication.

2. The multiplication by factors is not necessarily limited to the numbers of the multiplication table, *but may be extended to any composite number.* Thus instead of multiplying by 288, we may multiply by 12, 12, and 2, because $12\times 12\times 2=288$.

1. Multiply 213 by 18. The factors of 18 are 3 and 6, or 2 and 9. Since $3\times 6=18$, and $9\times 2=18$.

Therefore 213

3

639=prod. of 213×3.

6

3834=prod. of 639×6, which=213×18.

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64. What is a composite number? How are the numbers of the multiplication table produced? What are the smaller numbers called? To what may multiplication by factors be extended. Illustrate.

Also 213
9
1917=prod. of 213 by 9.
2
3834=prod. of 1917×2=213×18.

NOTE. Whenever the multiplier is composite, and susceptible of being divided into *more than one set of component parts*, it is immaterial which set of factors is taken since they each give the same product. Thus in the above sum the multiplier

18=6×3, or 9×2, or 3×3×2.

Each of which sets of factors produces the same result.

2. Multiply 6683 by 21. The factors of 21 are 3 and 7.

Therefore, 6683
3
20049=3 times 6683
7
140343=7 times 20049=7×3 times 6683.

3. Multiply 8975 by 24. *Ans.* 215400.

4. What would 759 acres of land cost at $15 an acre? *Ans.* $11385.

5. What would be the value of 1856 tons of hay at $16 per ton? *Ans.* $29696.

6. What would 27 horses cost at $95 each? *Ans.* $2565.

7. What would 36 yoke of oxen cost at $87 a yoke? *Ans.* $3132.

8. If a horse be worth $167, what is the value of 42 horses? *Ans.* $7014.

9. If 1 cow be valued at $18, what is the value of 35 cows? *Ans.* $630.

It is immaterial whether 18 be multiplied by 35, that is, by 7 and 5; or 35 be multiplied by 18, that is, by 3 and 6; for 18×7×5=$630; also 35×3×6=$630.

10. What is the value of 956 acres of land at $35 per acre? *Ans.* $33460.

11. How many bushels of wheat will 783 acres produce, if 1 acre produce 42 bushels? *Ans.* 32886 bushels.

When the multipliers can be divided into different sets of factors, what may be done, and with what result?

12. How far will a man travel in 49 days, if he travel at the rate of 93 miles a day? *Ans.* 4557 miles.

13. What would it cost to board 672 weeks at 28 shillings per week? *Ans.* 18816 shillings.

14. One hogshead contains 63 gallons. How many gallons in 859 hogsheads? *Ans.* 54117 gallons.

15. In 1 acre there are 160 square rods. How many square rods in 72 acres? *Ans.* 11520 sq. rd.

16. In 1 cubic foot there are 1728 inches. How many cubic inches in 64 cubic feet? *Ans.* 110592.

17. In 1 square foot there are 144 square inches. How many square inches in 56 square feet? *Ans.* 8064 sq. in. In 63 square feet? *Ans.* 9072.

18. In 1 hogshead there are 63 gallons. How many gallons in 21987 hogsheads? *Ans.* 1385181 gal.

19. How many gallons in 8536 hogsheads, if 1 hogshead contain 63 gallons? *Ans.* 537768 gal.

20. What is the cost of 359 tons of iron at $99 per ton? *Ans.* $35541.

§ **65.** To Multiply by any Number, whether Composite or not.

RULE.

Multiply each figure in the multiplicand separately by each figure of the multiplier, commencing with the right-hand figure of each, and place the several products under each other, with the right-hand figure of each separate product standing directly beneath the multiplying figure; then add the separate products.

Observe, 1st.—That the number of separate products *will equal the number of figures in the multiplier, unless some of those figures be ciphers; and if there be ciphers in the multiplier, then the number of separate products will be as many less than the number of figures in the multiplier as there are ciphers in that multiplier.*—2d. If there be one or more ciphers in the multiplier, for each cipher, *the fol-*

65. How multiply? How place the products? What is done with the separate products? How many separate products? With what exception? How write the separate products when ciphers occur in *the multiplier?*

lowing product must be carried one degree further to the left than it otherwise would be; that is, if there be no ciphers in the multiplier, *each succeeding product is written one degree further to the left, than the previous product;* if there be one cipher, *it is written two degrees,* and if two *contiguous* ciphers, *three degrees further, &c.*

1. What is the product of 7832 multiplied by 346?

PERFORMED.

7832
346

46992 = prod. of 7832 by 6 *units.*
31328 = " " " 4 *tens.*
23496 = " " " 3 *hundreds.*
2709872 = " " " 6 *units*+4 *tens*+3 *hundreds*=prod. of 7832×346.

The direct proof of multiplication is by division, and is consequently omitted here. We can, however, adopt the following indirect and less sure mode. Instead of multiplying by the figures of the multiplier *in the order of their local values,* (§ 5, 4,) *multiply in any other order, taking care to give each separate product a location corresponding to the local value of the multiplying figure.* That is, if we multiply first by the figure in the ten's place, *we must write the product of the unit figure one place to the right, and that of the hundreds one place to the left,* of the product of the tens; and in a corresponding order, if the figures of the multiplier are taken in any other than their natural order.

The above example proved:—

7832
346

31328 = prod. of 4 tens.
46992 = " " 6 units, one place to the right of tens
23496 = " " 3 hundreds, one place to left of tens.
2709872 = " " 346, the same as before.

How far to the left must each figure in the product be carried for each cipher in the multiplier? Explain. How can multiplication be proved? *Explain.*

Again,

```
    7832
     346
  -------
  23496   =prod. of 3 hundreds.
   46992= "  "  6 units, two places to right of hun.
  31328  = "  "  4 tens, one place to left of units.
  -------
  2709872= "  "  346, still the same.
```

We hence see that the direction in the rule, of commencing with the right-hand figure, is not *necessary for accuracy*, but *merely convenient.*

2. What is the product of 739 multiplied by 39 ?
Ans. 28821.
3. What is the product of 1642 multiplied by 243 ?
Ans. 399006.
4. What is the product of 3659 multiplied by 197 ?
Ans. 720823.
5. What is the product of 978 multiplied by 356 ?
Ans. 348168.
6. What is the product of 5849 multiplied by 871 ?
Ans. 5094479.
7. What is the product of 8567 multiplied by 462 ?
Ans. 3957954.
8. What is the product of 7768 multiplied by 369 ?
Ans. 2866392.
9. What is the product of 6793 multiplied by 842 ?
Ans. 5719706.
10. What is the product of 7183 multiplied by 279 ?
Ans. 2004057.
11. What is the product of 8956 multiplied by 357 ?
Ans. 3197292.
12. What is the product of 7947 multiplied by 435 ?
Ans. 3456945.
13. What is the product of 356 multiplied by 29 ?
Ans. 10324.
14. What is the product of 1487 multiplied by 453 ?
Ans. 673611.

Is it necessary to commence multiplying with the right-hand figure *of the multiplier?*

15. What is the product of 5371 multiplied by 287?
Ans. 1541477
16. What is the product of 1879 multiplied by 237?
Ans. 445323
17. What is the product of 4386 multiplied by 93?
Ans. 407898.
18. What is the product of 2957 multiplied by 298?
Ans. 881186.
19. What is the product of 2879 multiplied by 375?
Ans. 1079625.
20. What is the product of 3499 multiplied by 268?
Ans. 937732.
21. What is the product of 7846 multiplied by 654?
Ans. 5131284.
22. What is the product of 9347 multiplied by 374?
Ans. 3495778.
23. What is the product of 7953 multiplied by 733?
Ans. 5829549.

§ **66.** Perform the operations as indicated by the following signs.

1. $37896 \times 149 = 5646504$.
2. $874316 \times 5876 = 5137480816$.
3. $764379 \times 7186 = 5492827494$.
4. $3765738 \times 4567 = 17198125446$.
5. $6789345 \times 3892 = 26424130740$.
6. $6804673 \times 9872 = 67175731856$.
7. $778967 \times 5346 = 4164357582$.
8. $7832754 \times 3786 = 29654806644$.
9. $3456789 \times 5438 = 18798018582$.
10. $5676395 \times 7196 = 40847338420$.

The scholar should be required to enumerate each of the above products.

§ **67.** Whenever the multiplier consists of the figure 1 and ciphers, as 10, 100, 1000, 10000, the multiplication is performed *by writing the ciphers of the multiplier on the right of the multiplicand.*

67. How multiply by 10, 100, 1000, &c.

1. What is the product of 378 multiplied by 100 ?
Ans. 37800.
2. What is the product of 7536 multiplied by 1000 ?
Ans. 7536000.
3. What is the product of 8560 multiplied by 10 ?
Ans. 85600.
4. What is the product of 87469 multiplied by 10 ?
Ans. 874690.
5. What is the product of 66879 multiplied by 10000 ?
Ans. 668790000.
6. What is the product of 30807 multiplied by 100 ?
Ans. 3080700.

§ **68.** When the multiplier consists of a number larger than 1 with ciphers on its right, *multiply by the significant figures only, and place on the right of the product, ciphers equal in number to those of the multiplier.*

If there are ciphers both on the right of the multiplier and multiplicand, *multiply by the significant figures only, and place on the right of the product as many ciphers as are found in both.*

1. Multiply 46876 by 30. *Ans.* 1406280.

Multiply by the 3 only and write the cipher on the right of the product.

2. Multiply 67598 by 450. *Ans.* 30419100.
3. Multiply 879564 by 8100. *Ans.* 7124468400.
4. Multiply 897630 by 900. *Ans.* 807867000.
5. Multiply 78563 by 4900. *Ans.* 384958700.

§ **69.** When there are ciphers between the significant figures of the multiplier, *multiply by the significant figures only, and for each intervening cipher, remove the following product one degree further to the left.*

1. Multiply 50679 by 508.

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68. How multiply when the multiplier consists of a number larger than one, with ciphers annexed? What is to be done when there are ciphers on the right of both numbers?

69. *How is the* operation conducted when ciphers stand between *significant figures?*

PERFORMED.

50679
508
405432
253395 placed two degrees to the left because of the
25744932 cipher in the multiplier; as 255395 is the product of hundreds in the multiplier, its right-hand figure must stand in hundreds' place.

2. Multiply 87063 by 607. *Ans.* 52847241.
3. Multiply 9746904 by 609. *Ans.* 5935864636.
4. Multiply 739087 by 3009. *Ans.* 2223912783.
5. Multiply 7896504 by 6002. *Ans.* 47394817008.

§ **70.** To multiply by any number of 9s. One 9 is 1 less than 10; two 9s are 1 less than 100; three 9s are 1 less than 1000, &c.; that is, 9=10−1; 99=100−1; 999=1000−1, &c. Hence if any number is to be multiplied by 9, and a cipher is placed on its right, (§ **68**,) the result is the multiplicand taken once more than the required number of times; therefore from that result substract the *given multiplicand*, and the result will be the product of the multiplicand multiplied by 9. The same explanation is applicable to any number of 9s. Therefore when the multiplier consists of 9s only, *place as many ciphers on the right of the multiplicand as there are* 9s *in the multiplier, and from the result subtract the multiplicand.*

The same mode of procedure may be adopted whenever the multiplier is 98, 998, &c., or 97, 997, &c., *by writing the ciphers as before, and subtracting from the result two, three, or more times the multiplicand, as the case may require.*

1. Multiply 7958 by 9999.

PERFORMED.

79580000 Four ciphers added because the multiplier
7958 contains four 9s.
79572042=prod. of 7958×9999.

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70. How multiply by 9? How multiply by a number of 9s? *What is said of 98, 97, &c.?*

2. Multiply 53874 by 99.

PERFORMED.

5387400 Two ciphers added because the multiplier
53874 contains two 9s.

5333526=prod. of 53874×99.

3. Multiply 85674 by 999. *Ans.* 85588326.
4. Multiply 73218 by 9999. *Ans.* 732106782.
5. Multiply 763978 by 9. *Ans.* 6875702.
6. Multiply 54789 by 99. *Ans.* 5424111.
7. Multiply 81567 by 98.

PERFORMED.

8156700=multiplicand multiplied by 100.
81567×2= 163134 " " " 2.
7993566 " " " 98.

8. Multiply 76358 by 997.

PERFORMED.

76358000=multiplicand multiplied by 1000.
229074= " " " 3.
76128926= " " " 997.

9. Multiply 46809 by 99. *Ans.* 4634091.
10. Multiply 83964 by 999. *Ans.* 83880036.
11. Multiply 586738 by 97. *Ans.* 56913586.
12. Multiply 78596 by 998. *Ans.* 78438808.
13. Multiply 846709 by 98. *Ans.* 82977482.
14. Multiply 973875 by 997. *Ans.* 970953375.

GENERAL APPLICATION OF MULTIPLICATION.

§ 71.—1. What would 693 pounds of sugar cost at 14 cents a pound? *Ans.* 9702 cents.

The answer to the previous sum is given in cents, and since 100 cents make a dollar, there will be a dollar for every hundred cents in 9702 cents. Now if we enumerate this number, (9702,) we find the 7 is hundreds, and the 9, thousands, or tens of hundreds. Hence there are 97 hun-

71. *How many* cents make a dollar? How many dollars in 9702 *cents? How many* hundreds in 9702?

dred and 2 cents, or 97 dollars and 2 cents. To designate dollars from cents we place a period between the figures indicating cents and those indicating dollars, and since the value of 1 dollar is 100 cents, any value in dollars is reduced to cents by placing two ciphers on the right of the dollars, (§ **67**;) and consequently any value in cents is expressed in dollars, or dollars and cents by cutting two figures from the right of the number expressing that value. The figures to the *right of the point* will be *cents*, and those *on the left* will be *dollars*. Thus the answer to the previous sum, viz., 9702 cents, is reduced to dollars by placing a period between the 7 and the cipher, thus, 97.02, and is read 97 dollars and .02 (two) cents. Any product, thus expressing cents should be reduced to dollars, whenever it is larger than 100, and the appropriate sign ($) for dollars placed before the number. 9702 cents, when reduced to dollars, is written $97.02, and read 97 dollars and 2 cents.

2. What would 463 tons of hay cost at $18.64 a ton?

Ans. $8630.32.

In multiplication, one of the given numbers determines how many times the other is to be repeated. Thus, in the above sum, if 1 ton of hay cost dollars, any number of tons will likewise cost dollars; consequently the dollars are to be repeated as many times as there are single tons in 463 tons. Hence $18.64 is properly the multiplicand, or number to be repeated, while 493 is the multiplier determining the number of repetitions. It is usual, however, to make the larger number the multiplicand, and the smaller one, the multiplier, it being more convenient; while the character or value of the product is not affected in the least by the order in which the numbers are taken.

3. What will 897 pairs of shoes cost at $1.75 a pair?

Ans. $1569.75.

How designate dollars from cents? How is any value in dollars reduced to cents? How is any value in cents reduced to dollars? What figures are cents and what dollars? How reduce 9702 cents to dollars? How read? When should any product in cents be reduced to dollars? What sign marks dollars? What is determined by one of the numbers in multiplication? Illustrate. Which number is the proper multiplicand? Which the multiplier? What is usual?

4. What will 398 gallons of molasses cost at 56 cents per gallon? *Ans.* $222.88.

5. What is the value of 296 yards of cloth at $4.75 per yard? *Ans.* $1406.00.

6. What will 763 bushels of wheat cost at $1.75 a bushel? *Ans.* $1335.25.

7. What will a drove of 89 cows cost at an average price of $18.56? *Ans.* $1651.84.

8. Sold 1789 pounds of butter at 18 cents a pound. How much did I receive? *Ans.* $322.02.

9. There are 48 lines on a page and 58 letters in each line, how many letters on a page? *Ans.* 2279 letters.

10. If one box of raisins be worth $2.16, what will 737 boxes cost? *Ans.* $1591.92.

11. If one bushel of corn be worth 78 cents, what is the value of 839 bushels? *Ans.* $654.42.

12. If a man travel 586 miles per day, how far will he travel in 369 days? *Ans.* 216234 miles.

13. If a laborer receive $342 for a year's labor, how much would he earn in 17 years? *Ans.* $5814.

14. What cost 269 tons of hay at $16.75 per ton? *Ans.* $4505.75

15. What will 397 boxes of raisins come to at $3.25 a box? *Ans.* $1290.25.

16. What will 37 yoke of oxen cost at an average of $73.87 a pair? *Ans.* $2733.19.

17. What is the value of 64980 pounds of gunpowder at 33 cents a pound? *Ans.* $21443.40.

18. What is the value of 1653 pounds of chocolate at 17 cents a pound? *Ans.* $281.01.

19. What is the value of 615 umbrellas at $2.18 each? *Ans.* $1340.70.

20. What would 5278 trunks cost at $5.37 each? *Ans.* $28342.86.

21. What would 896 arithmetics cost at 42 cents each? *Ans.* $376.32.

22. What would 1728 pounds of morocco cost at 29 cents a pound? *Ans.* $501.12.

23. What would 34796 pounds of wax cost at 24 cents *a pound?* *Ans.* $8351.04.

MERCHANTS' AND TRADESMENS' BILLS.

§ 72. To find the total value of a bill of goods:—

Find the value of each item of said bill by multiplying the price per article by the number of articles, and add the several values together.

Find the amount of the several articles specified in the following bills:—

1.

J. Norton, Esq.

Bought of Samuel Brown,

5 pairs of boots, at $3.25 a pair, - - - - - -	$16.25
6 yards of broadcloth, at $4.50 a yard, - - -	27.00
12 skeins of sewing silk, at 5 cents a skein, - -	.60
16 yards of muslin, at 35 cents a yard, - - -	5.60
	$49.45

Received payment,

Samuel Brown.

Jan, 17, 1849.

2.

John Hutton, Esq.

Bought of James Fisher,

27 pounds of coffee, at 11 cents a pound, - - -	$2.97
16 pounds of sugar, at 9 cents a pound, - - - -	1.44
21 pounds of raisins, at 7 cents a pound, - - -	1.47
18 pounds of butter, at 21 cents a pound, - - -	3.78
	$9.66

Received payment,

James Fisher.

Jan. 18, 1849.

3.

George Smith

Bought of Samuel Johnson,

37 yards of silk, at $1.25 a yard, - - - - - -	$
32 yards of muslin, at 52 cents a yard, - - -	
12 pairs of shoes, at $1.82 a pair, - - - - -	
16 hats, at $2.18 a hat, - - - - - - - - -	
21 pairs of shoes, at $2.10 a pair, - - - - -	

Required the amount of the bill. *Amt.* $163.71

4.

James Rankin

Bought of Johnson & Co.

30 barrels of flour, at $5.20 a barrel, - - - - $
27 barrels of beef, at $8.75 a barrel, - - -
16 barrels of pork, at $8.12 a barrel, - - -
42 kegs of lard, at $2.90 a keg, - - - - -

Required the total value. *Amt.* $643.97

5.

T. Jenkins, Esq.

Bought of Smith & Stodard,

37 bushels of wheat, at $1.75 per bushel, - - $
42 bushels of corn, at $0.92 per bushel, - - -
24 bushels of rye, at $0.75 per bushel, - - -
89 bushels of oats, at $0.33 per bushel, - - -

Amt. $150.76

Received payment,

Smith & Stodard.

6.

R. Prince, Esq.

Bought of James Sanford & Co.

32 arithmetics, at 29 cents, - - - - - - - - $
61 readers, at 35 cents, - - - - - - - - -
88 spellers, at 39 cents, - - - - - - - - -
28 geographies, at 56 cents, - - - - - - -

Amt. $80.63

Received payment,

J. Sanford & Co.

7.

Samuel Jones

Bought of Richard Lee & Co.

54 Watts's Psalms and Hymns, at 42 cents, - - $
29 New Testaments, at 63 cents, - - - - - -
47 Watts on the Mind, at 27 cents, - - - - -
59 New England primers, at 15 cents, - - -
26 Doddridge's Rise and Progress, at 18 cents, -

Amt. $67.17

Received payment,

Richard Lee & Co.

8.

P. Stevenson

Bought of George Knowlton,

356 pounds of butter, at $0.17 a pound, - - $
287 pounds of lard, at $0.12 a pound, - - -
99 pounds of cheese, at $0.13 a pound, - -
287 pounds of mutton tallow, at $0.07 a pound,
188 pounds of beef tallow, at $0.10 a pound, -

Amt. $146.72

Received payment,

G. Knowlton.

9.

Robert Baker

Bought of James Snedeker,

18 yoke of oxen, at $87 per yoke, - - - - $
24 span of horses, at $156 per span, - - - -
72 cows, at $27 each, - - - - - - - - -

Amt. $7264

Received payment,

James Snedeker.

10.

Joseph Stewart, Esq.

Bought of J. H. Jones,

17 yards superfine broadcloth, at $5.80 per yard, $
42 yards muslin, at 32 cents per yard, - - - -
63 yards black silk, at 89 cents per yard, - - -
13 vest patterns, each 75 cents, - - - - - - -
108 pairs of cotton hose, at 19 cents each, - - -

Amt. $198.38

11.

G. Godfrey, Esq.

Bought of Seth Miller & Co.

33 hundredweight of refined sugar, at $8.37 - $
28 hundredweight of crushed sugar, at $7.92 -
63 bags of Java coffee, at $3.20 per bag, - -
9 chests of tea, at $18.63 per chest, - - - -
12 barrels of molasses, at $9 per barrel, - - -

Amt. $976.24

12.

Marcus H. Stoddard

Bought of Samuel C. Pike & Co.

3 hogsheads of molasses, each 63 gallons, at 27 cents a gallon, - - - - - - - - - - - - $
7 casks of vinegar, each 42 gallons, at 30 cents a gallon, - - - - - - - - - - - - -
8 quintals of codfish, each 112 pounds, at 5 cents a pound, - - - - - - - - - - - - -
36 hams, each 18 pounds, at 9 cents a pound, -
39 hams, each 16 pounds, at 7 cents a pound, -

Amt. $286.03

Received payment,

Samuel C. Pike & Co.

13.

S. Van Rensselaer

Bought of Samuel Sniffen,

3 dozen mahogany chairs, at $5 each, - - - $
12 dozen cane-seated chairs, at $3.20 each, - -
8 dozen kitchen chairs, at $1.73 each, - - -
6 bureaus, at $17.86 each, - - - - - - -

Amt. $914.04

Received payment,

Samuel Sniffen.

14.

George Grant, Esq.

Bought of Benj. Butler,

3 boxes of sugar, each 108 pounds, at 12 cents a pound, - - - - - - - - - - - - - $
3 hogsheads of molasses, each 63 gallons, at 36 cents a gallon, - - - - - - - - - - -
5 chests of tea, each 96 pounds, at 84 cents a pound, - - - - - - - - - - - - - -
6 casks of rice, each 280 pounds, at 5 cents a pound, - - - - - - - - - - - - - -

Amt. $594.12

Received payment,

Benj. Butler.

15.

Samuel Dean, Esq.

Bought of Sylvester Crane & Co.

89 bags of coffee, each 96 pounds, at 12 cents a pound, - - - - - - - - - - - - - - - - $

36 chests of tea, each 63 pounds, at 69 cents a pound, - - - - - - - - - - - - - - -

8 bales of sheeting, each 64 yards, at 14 cents a yard, - - - - - - - - - - - - - - -

5 boxes of sugar, each 256 pounds, at 8 cents a pound, - - - - - - - - - - - - - - -

14 boxes of raisins, each 18 pounds, at 13 cents a pound, - - - - - - - - - - - - - -

Amt. $2797.04

16.

R. Covil, Esq.

Bought of Sanford Spalding,

17 hogsheads of molasses, each 63 gallons, at 27 cents a gallon, - - - - - - - - - - - - $

13 barrels of vinegar, each 36 gallons, at 18 cents a gallon, - - - - - - - - - - - - -

16 boxes of sugar, each 178 pounds, at 7 cents a pound, - - - - - - - - - - - - - - -

23 kegs of lard, each 35 pounds, at 11 cents a pound, - - - - - - - - - - - - - - - -

Amt. $661.32

17.

J. Dunham, Esq.

Bought of Seth Osborn,

18 pounds of tea, at 56 cents a pound, - - - - $

27 pounds of coffee, at 14 cents a pound, - - -

48 pounds of sugar, at 11 cents a pound, - - -

21 pounds of raisins, at 15 cents a pound, - - -

31 pounds of butter, at 21 cents a pound, - - -

Amt. $28.80

Received payment,

Seth Osborn.

DIVISION.

§ 73. By Division we ascertain how many times one number contains another. The sign of divison is thus written ÷. The left-hand number is to be divided by that on the right.

DIVISION TABLE.

1÷1= 1	2÷2= 1	3÷3= 1	4÷4= 1
2÷1= 2	4÷2= 2	6÷3= 2	8÷4= 2
3÷1= 3	6÷2= 3	9÷3= 3	12÷4= 3
4÷1= 4	8÷2= 4	12÷3= 4	16÷4= 4
5÷1= 5	10÷2= 5	15÷3= 5	20÷4= 5
6÷1= 6	12÷2= 6	18÷3= 6	24÷4= 6
7÷1= 7	14÷2= 7	21÷3= 7	28÷4= 7
8÷1= 8	16÷2= 8	24÷3= 8	32÷4= 8
9÷1= 9	18÷2= 9	27÷3= 9	36÷4= 9
10÷1=10	20÷2=10	30÷3=10	40÷4=10
11÷1=11	22÷2=11	33÷3=11	44÷4=11
12÷1=12	24÷2=12	36÷3=12	48÷4=12
5÷5= 1	6÷6= 1	7÷7= 1	8÷8= 1
10÷5= 2	12÷6= 2	14÷7= 2	16÷8= 2
15÷5= 3	18÷6= 3	21÷7= 3	24÷8= 3
20÷5= 4	24÷6= 4	28÷7= 4	32÷8= 4
25÷5= 5	30÷6= 5	35÷7= 5	40÷8= 5
30÷5= 6	36÷6= 6	42÷7= 6	48÷8= 6
35÷5= 7	42÷6= 7	49÷7= 7	56÷8= 7
40÷5= 8	48÷6= 8	56÷7= 8	64÷8= 8
45÷5= 9	54÷6= 9	63÷7= 9	72÷8= 9
50÷5=10	60÷6=10	70÷7=10	80÷8=10
55÷5=11	66÷6=11	77÷7=11	88÷8=11
60÷5=12	72÷6=12	84÷7=12	96÷8=12

78. What is ascertained by division?

9÷9= 1	10÷10= 1	11÷11= 1	12÷12= 1
18÷9= 2	20÷10= 2	22÷11= 2	24÷12= 2
27÷9= 3	30÷10= 3	33÷11= 3	36÷12= 3
36÷9= 4	40÷10= 4	44÷11= 4	48÷12= 4
45÷9= 5	50÷10= 5	55÷11= 5	60÷12= 5
54÷9= 6	60÷10= 6	66÷11= 6	72÷12= 6
63÷9= 7	70÷10= 7	77÷11= 7	84÷12= 7
72÷9= 8	80÷10= 8	88÷11= 8	96÷12= 8
81÷9= 9	90÷10= 9	99÷11= 9	108÷12= 9
90÷9=10	100÷10=10	110÷11=10	120÷12=10
99÷9=11	110÷10=11	121÷11=11	132÷12=11
108÷9=12	120÷10=12	132÷11=12	144÷12=12

§ 74. MENTAL EXERCISES IN DIVISION.

1÷1 are how many?		2÷2 are how many?	
3÷1	" "	6÷2	" "
2÷1	" "	4÷2	" "
5÷1	" "	8÷2	" "
4÷1	" "	12÷2	" "
7÷1	" "	10÷2	" "
6÷1	" "	14÷2	" "
9÷1	" "	18÷2	" "
8÷1	" "	16÷2	" "
10÷1	" "	20÷2	" "
12÷1	" "	24÷2	" "
11÷1	" "	22÷2	" "

3÷3 are how many?		4÷4 are how many?	
9÷3	" "	12÷4	" "
6÷3	" "	8÷4	" "
12÷3	" "	16÷4	" "
18÷3	" "	24÷4	" "
15÷3	" "	20÷4	" "
21÷3	" "	28÷4	" "
27÷3	" "	36÷4	" "
24÷3	" "	32÷4	" "
30÷3	" "	40÷4	" "
36÷3	" "	48÷4	" "
33÷3	" "	44÷4	" "

5÷5 are how many?	6÷6 are how many?
15÷5 " "	18÷6 " "
10÷5 " "	12÷6 " "
20÷5 " "	24÷6 " "
30÷5 " "	36÷6 " "
25÷5 " "	30÷6 " "
35÷5 " "	42÷6 " "
45÷5 " "	54÷6 " "
40÷5 " "	48÷6 " "
50÷5 " "	60÷6 " "
60÷5 " "	72÷6 " "
55÷5 " "	66÷6 " "

7÷7 are how many?	8÷8 are how many?
21÷7 " "	24÷8 " "
14÷7 " "	16÷8 " "
28÷7 " "	32÷8 " "
42÷7 " "	48÷8 " "
35÷7 " "	40÷8 " "
49÷7 " "	56÷8 " "
63÷7 " "	72÷8 " "
56÷7 " "	64÷8 " "
70÷7 " "	80÷8 " "
84÷7 " "	96÷8 " "
77÷7 " "	88÷8 " "
91÷7 " "	104÷8 " "

9÷9 are how many?	10÷10 are how many?
27÷9 " "	30÷10 " "
18÷9 " "	20÷10 " "
36÷9 " "	40÷10 " "
54÷9 " "	60÷10 " "
45÷9 " "	50÷10 " "
63÷9 " "	70÷10 " "
81÷9 " "	90÷10 " "
72÷9 " "	80÷10 " "
90÷9 " "	100÷10 " "
108÷9 " "	120÷10 " "
99÷9 " "	110÷10 " "
117÷9 " "	130÷10 " "

11÷11	are how many?		12÷12	are how many?	
33÷11	"	"	36÷12	"	"
22÷11	"	"	24÷12	"	"
44÷11	"	"	48÷12	"	"
66÷11	"	"	72÷12	"	"
55÷11	"	"	60÷12	"	"
77÷11	"	"	84÷12	"	"
99÷11	"	"	108÷12	"	"
88÷11	"	"	96÷12	"	"
110÷11	"	"	120÷12	"	"
132÷11	"	"	144÷12	"	"
121÷11	"	"	132÷12	"	"

§ 75. Further Application of Division Table.

1.	4÷ 2	are how many?	26.	81÷ 9	are how many?
2.	8÷ 4	" "	27.	88÷11	" "
3.	12÷ 3	" "	28.	54÷ 9	" "
4.	10÷ 5	" "	29.	84÷12	" "
5.	15÷ 5	" "	30.	18÷ 9	" "
6.	12÷ 4	" "	31.	72÷ 8	" "
7.	16÷ 4	" "	32.	96÷ 8	" "
8.	18÷ 9	" "	33.	96÷12	" "
9.	16÷ 8	" "	34.	88÷ 8	" "
10.	24÷ 8	" "	35.	63÷ 9	" "
11.	24÷ 3	" "	36.	99÷ 9	" "
12.	24÷ 6	" "	37.	132÷12	" "
13.	24÷ 4	" "	38.	121÷11	" "
14.	27÷ 9	" "	39.	144÷12	" "
15.	28÷ 7	" "	40.	96÷12	" "
16.	30÷10	" "	41.	72÷ 8	" "
17.	32÷ 8	" "	42.	81÷ 9	" "
18.	28÷ 4	" "	43.	42÷ 6	" "
19.	21÷ 7	" "	44.	108÷12	" "
20.	42÷ 7	" "	45.	84÷ 7	" "
21.	63÷ 9	" "	46.	88÷11	" "
22.	54÷ 9	" "	47.	77÷ 7	" "
23.	49÷ 7	" "	48.	56÷ 8	" "
24.	56÷ 8	" "	49.	84÷12	" "
25.	72÷ 9	" "	50.	63÷ 9	" "

§ 76.—1. How many pairs of boots can be bought for $24, at $3 a pair? (24÷3=how many?).

2. How many quarts of milk, at 4 cents a quart, can be bought for 16 cents? How many for 32 cents? for 24 cents? for 40 cents? for 48 cents?

3. How many pounds of cheese can be bought for 40 cents, at 8 cents a pound? for 32 cents? for 64 cents? for 56 cents? for 72 cents? for 96 cents? for 24 cents?

4. How long will it require to travel 49 miles at 7 miles an hour? 63 miles? 77 miles? 84 miles? 35 miles? 42 miles? 21 miles?

5. How long will 35 bushels of oats suffice for a horse, if he eat 5 bushels a week? How long will 45 bushels suffice? 60 bushels? 55 bushels? 30 bushels?

6. How long will it require to travel 72 miles at 9 miles an hour? how long to travel 108 miles? 99 miles? 54 miles? 45 miles? 63 miles?

7. If a barrel of molasses cost $11, how many barrels can be bought for $33? for $77? for $99? for $55? for $88?

8. If a man earn $8 in 1 day, in how many days will he earn $48? $64? $96? $40? $32? $72?

9. If a pound of sugar cost 9 cents, how many pounds can be bought for 81 cents? for 90 cents? for 45 cents? for 54 cents? for 63 cents?

10. If a man spend $12 a week, in how many weeks will he spend $96? $132? $72? $84? $60? $144?

11. At 4 cents a mile, how far can I ride for 48 cents? 32 cents? 16 cents? 20 cents? 40 cents? 44 cents?

12. If it cost 7 cents to ride 1 mile, how many miles can I ride for 63 cents? for 84 cents? for 56 cents? for 35 cents? for 77 cents?

13. How many shillings are there in 72 pence, 12 pence being required to make 1 shilling? in 96 pence? in 108 pence? in 132 pence? in 60 pence?

14. If a pound of lard cost 11 cents, how many pounds may be bought for 110 cents? for 132 cents? for 88 cents? for 99 cents? for 55 cents?

15. Bought a sheep for $6. How many sheep can I buy for $54? for $66? for $72? for $48? for $36? for $42?

16. If 8 shillings make 1 dollar, how many dollars are there in 64 shillings? in 88 shillings? in 56 shillings? in 96 shillings? in 72 shillings?

17. There are 12 ounces in 1 pound Troy. How many pounds in 72 ounces? in 48 ounces? in 84 ounces? in 60 ounces? in 36 ounces?

§ **77.** It often happens that the dividing number is not an exact measure of the one to be divided; that is, after the division is performed, *a portion of the divided number, less than the divisor, remains.* This is called the *remainder.* The following is an illustration. Suppose 23 is to be divided by 5. The divisor 5 is not contained in 23 so many as 5 times, since 5 times 5 are 25, a number larger than 23. It is, however, contained in 23, 4 times, since 4 times 5 are 20; a number less than 23 by 3. Hence 5 is contained in 23, 4 times and 3 remainder.

1. How many times is 6 contained in 39, and what is the remainder?

2. How many times is 4 contained in 18, and what is the remainder?

3. A man having $37, bought cloth at $5 a yard. How many yards could he buy, and how many dollars would he have left?

4. A boy having 47 cents, bought melons at 6 cents each. How many melons could he buy, and how many cents have left?

5. A scholar has 51 sums to perform. During how many hours can he perform 7 per hour, and how many will remain?

6. If a pound of raisins cost 7 cents, how many pounds can I buy for 60 cents, and how many cents left?

7. If a pound of tamarinds cost 6 cents, how many pounds can be bought for 70 cents, and how many cents left? for 49 cents? for 59 cents?

8. If a pound of dates cost 12 cents, how many pounds can be bought for 140 cents, and how many cents left?

9. If a barrel of flour cost $9, how many barrels can be bought for $70? for $88? for $49? for $57? for $68?

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77. What often happens? Illustrate?

10. How many times 4 in 33, and how many remain? in 38? in 46?
11. How many times 8 in 46, and how many remain? in 54? in 71?
12. How many times 11 in 37, and how many remain? in 49? in 93?
13. How many times 12 in 52, and how many remain? in 79? in 101?
14. How many times 9 in 61, and how many remain? in 86? in 107?
15. How many times 7 in 51, and how many remain? in 83? in 78?
16. How many times 6 in 39, and how many remain? in 57? in 47?
17. How many times 12 in 79, and how many remain? in 63? in 95?
18. How many times 11 in 81, and how many remain? in 69? in 98?
19. How many times 10 in 64, and how many remain? in 73? in 117?

SIMPLE DIVISION.—*For the Slate.*

§ **78.** From the previous questions and answers we learn the nature of Division; viz., that it is a process of ascertaining *how many times one number contains another.*

Two numbers are given, one of which is to be divided by the other. That *to be divided* is called the *dividend;* that by which we divide, is called the *divisor;* and if, after the operation is performed, a number be left undivided, that is called the *remainder.* The *quotient,* or number, *obtained by dividing the dividend by the divisor, is one of the factors,* (§ **64.** 1,) *or equal parts into which the dividend is separated.* The number of these factors *equals the units*

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78. What do we ascertain by division? How many numbers are given? What is that to be divided called? What that by which we *divide?* What call the number left undivided? What is the quo*tient?*

in the divisor; hence if the *quotient be written down a number of times equal to the units composing the entire divisor,* and their sum obtained, it will *exactly correspond with the dividend* in all instances, *except when there is a remainder;* if a remainder exist, that must likewise be added to the amount of factors.

ILLUSTRATION. If we divide 20 by 4 we obtain 5 as a quotient; 4 and 5 are therefore the factors of 20; and since the divisor 4 contains four units, the quotient 5 must be repeated 4 times to produce the dividend 20; thus $5+5+5+5=20$. Again, $23\div4=5$ and 3 remainder, and $5+5+5+5+3=23$; that is, the four factors into which 23 is divided, and the number (3) which remains over and above these four factors, being added, produce the dividend 23.

It is obvious that the remainder must in every instance be less than the divisor, for were it larger than the divisor, the dividend would contain that divisor one time more than is indicated.

At paragraph 73, we gave the character $\div$, as that used to imply division. It is also expressed by writing the dividend over the divisor with a short line between them; thus $\frac{27}{3}$ implies that 27 is to be divided by 3.

The dividend may be a *mere abstract number*, or a number *representing some particular objects*, or *valuation*. The *divisor* is always to be regarded as *merely a number, showing the number of equal factors or quantities into which the dividend is to be resolved;* while the quotient, being one of the equal parts into which the dividend is resolved, must be of the same name or kind as the dividend. The remainder being also an undivided part of the dividend, agrees with it in kind. Division is the reverse of Multiplication, and is proved by it. The divisor showing the *number* of factors

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What is a factor? (§ 64. 1.) The number of these factors equals what? How proved? What exception? How then? Give illustrations? How does the remainder compare with the divisor? Why? What is the sign of division? How else expressed? What may the dividend be? The divisor what? Showing what? The quotient must agree in kind with what? The remainder with what? How is division proved? What does the divisor show?

obtained from the dividend, and the quotient being one of these factors, the product of these two terms necessarily produces the dividend in all cases except where there is a remainder. If a remainder exist, it must be added to the product of the divisor and quotient to produce the dividend.

Division is usually separated into two sections: the first embracing all cases when the divisor is not more than 12; the second, all cases when the divisor is more than 12.

CASE I.

§ 79. The following rule is to be used only when the divisor does not exceed 12.

RULE.

Write down the dividend, and placing a short curve line on the left, write the divisor at the left of that curve, and draw a horizontal line directly beneath the dividend.

Take from the left of the dividend the smallest number of figures capable of containing the divisor, and observe how many times it is contained, and whether there be any remainder, writing the figure expressing the times, beneath the line, directly under the figure or figures divided. If there be a remainder, write the following figure of the dividend on the right of that remainder, and divide as before, placing the figure obtained, under the dividend figure last taken. Continue this process, till all the figures of the dividend have been taken; the number obtained will be the quotient.

If the figure or figures taken divide without remainder, proceed with the remaining figures as before; and if in assuming any figure of the dividend, the number obtained

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What is the quotient? Their product, what? What exception? How, if there be a remainder? How many sections? The first, what? The second, what?

79. What is the rule? How write the sum? How many figures are taken, and where? Where write the quotient figure? If there be a remainder, what is done? Where place the figure obtained? How long continue the process? If the figures divide without remainder, how proceed? When place a cipher in the quotient?

be less than the divisor, place a cipher in the quotient, and, assuming another figure of the dividend, proceed as before.

1. Divide 9636 by 3. PERFORMED. 3)9636
   3212

The work here is very simple, as each figure of the dividend contains the divisor one or more times without remainder.

2. Divide 84884 by 4. *Ans.* 21221.
3. Divide 684264 by 2. *Ans.* 342132.

The following sums will require more care. In the first place, the left-hand figure of the dividend is in most cases *less than the divisor*, so that it may be necessary to take *two or even three* of the left-hand figures of the dividend to obtain a number sufficient to contain the divisor; and in the second place, the figure or figures divided, will divide without remainder only occasionally, so that instead of dividing each figure separately, we must write them successively on the right of the remainders of a previous division, and divide the number thus produced.

OPERATION.

4. Divide 1728 by 4. 4)1728=dividend.
   432=quotient.

Here the left-hand figure 1 will not contain the divisor 4, and we therefore divide the 17, that is, we assume the two left-hand figures. The divisor 4 is contained in 17, 4 times and 1 over. The 4 is written down and 2, the third figure of the dividend, is placed on the right of the remainder 1, and the number 12 obtained. This number contains the divisor 4, 3 times and no remainder, and 8 contains the divisor 2 times and no remainder. Hence the required quotient is 432.

It is important to know the precise numerical value of the quotient figures. That value is in all cases the same as that

How does the left-hand figure of the dividend often compare with the divisor? How many figures of the divisor may be taken at a time? What is a second difficulty? What is the value of each figure in any quotient?

of the figure or figures divided. In the above sum, had the left-hand figure, 1, been sufficient to contain the divisor, the figure obtained by dividing would have been thousands, because the 1 occupies the place of thousands. But as it is we take the two left-hand figures, viz., 17, which are hundreds, and thus obtain 4 hundred as the left-hand figure of the quotient. When we take the 2 of the dividend, that being tens, we obtain tens, and the 8 being units, gives units. It will be observed that when two figures are divided together, it is *the right-hand one*, that determines the denomination.

OPERATION.

5. Divide 3568 by 8. $8\overline{)3\,5\,6\,8}$=dividend.
$4\,4\,6$=quotient.

EXPLANATION. 3, the left-hand figure, will not contain the divisor, we therefore take 35, which contains 8, 4 times and 3 over; then placing the 6 on the right of the 3 remaining, we obtain 36, and this (36) contains 8, 4 times and 4 over; then, lastly, the 8 of the dividend placed on the right of the remainder 4, forms the number 48, which contains the divisor 6 times. We thus obtain 446 as the quotient. The 35 of the dividend is hundreds, and consequently the left-hand 4 is hundreds, the second 4 is tens, and the 6, units; each corresponding in denomination with the figure of the dividend from which it is obtained?

6. Divide 34984 by 4. *Ans.* 8746.
7. Divide 40770 by 6. *Ans.* 6795.
8. Divide 471784 by 8. *Ans.* 58973.
9. Divide 296802 by 9. *Ans.* 32978.
10. Divide 413248 by 11. *Ans.* 37568.
11. Divide 1016868 by 12. *Ans.* 84739.
12. Divide 59927 by 7. *Ans.* 8561.
13. Divide 437675 by 5. *Ans.* 87535.

In the above sums no remainder is found. The following are each attended with a remainder.

14. Divide 769537 by 6. *Ans.* 128256, and 1 rem.

The remainder is of the same denomination as the dividend *or number* divided.

15. Divide 6837981 by 11. *Ans.* 621634, and 7 rem.
16. Divide 778891 by 8. *Ans.* 97361, and 3 rem.
17. Divide 9867310 by 12. *Ans.* 822275, and 10 rem.
18. Divide 3897150 by 7. *Ans.* 556735, and 5 rem.
19. Divide 87639577 by 8. *Ans.* 10954947, and 1 rem.
20. Divide 4687931 by 9. *Ans.* 520881, and 2 rem.
21. Divide 4396178 by 9. *Ans.* 488464, and 2 rem.

§ 80. In the following sums, divide each dividend by each of the given divisors separately, and add the quotients and remainders.

1. Divide 783596 by 5 and 8.

OPERATION. 783596÷5=156719, and 1 rem.
783956÷8= 97994, and 4 rem.
Ans. 254713, and 5 rem.

2. Divide 346652 by 4 and 6. *Ans.* 144438, and 2 rem.
3. Divide 7418679 by 3 and 7. *Ans.* 3532704, and 2 rem.
4. Divide 687054 by 7 and 9. *Ans.* 174489, and 7 rem.
5. Divide 963785 by 6 and 8. *Ans.* 281103, and 6 rem.
6. Divide 4567811 by 2 and 11. *Ans.* 2699160, and 7 rem.
7. Divide 854673031 by 5 and 9.
Ans. 265898276, and 2 rem.
8. Divide 436958 by 6 and 10. *Ans.* 116521, and 10 rem.
9. Divide 734697 by 3 and 12. *Ans.* 306123, and 9 rem.

10. If 8 drams make one ounce, apothecaries' weight, how many ounces in 57893 drams?
Ans. 7236 ounces, and 5 drams over.

11. If it cost 7 cents to ride 1 mile, how far can I ride for 6978 cents? *Ans.* 996 miles, and 6 cents over.

12. If an acre of land is worth $11, how many acres can be bought for $986744. *Ans.* 89704 acres.

13. If I travel 12 miles daily, in how many days can I travel 9867540 miles? *Ans.* 822295 days.

14. If a locomotive run 12 miles in 1 hour, in how many hours will it run 10368? *Ans.* 864 hours.

15. How many coats, at $9 each, can be bought for $187353? *Ans.* 20817 coats.

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80. How are the following sums to be divided?

CASE II.

§ 81. The following is the rule for dividing when the divisor is any number larger than 12.

RULE.

Write down the dividend, and, drawing a short curve both on its right and left, place the divisor on the left.

Take from the left of the dividend the smallest number of figures capable of containing the divisor; and, observing how many times it is contained in those figures, place the figure expressing the number of times on the right of the dividend, as the first quotient figure.

Multiply the divisor by the quotient figure, and place the product under the figures divided.

Subtract this product from the figure or figures divided, and place on the right of the remainder the next left-hand figure of the dividend.

Divide the number thus obtained by the given divisor, and place the figure obtained, as the second figure of the quotient.

Continue the process of dividing, multiplying, subtracting, and bringing down the figures of the dividend, till all are divided; the figures of the quotient will express the times the divisor is contained in the dividend.

If, after the division is completed, an undivided number be left, that is a remainder.

If, on bringing down a figure of the dividend to the remainder, the number produced be less than the divisor, a cipher must be placed in the quotient, and another figure of the dividend brought down.

If in any instance, after the product of the quotient and divisor has been subtracted, *a number greater than the divisor* be left, the quotient figure is *too small*; but if that product *be greater than the number divided*, it is *too large*.

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81. Case 2, embraces what? How write the dividend, &c.? The divisor? Take how many figures? Observe what? How multiply? What and how subtract? How divide? Continue what? What call any undivided number, after the division is completed? When place a cipher in the quotient? How can we know if the quotient *figure be too small*? too large?

1. Divide 15341 by 29.

PERFORMED.
```
29)15341(529=quotient.
   145
   ---
    84
    58
   ---
   261
   261
   ---
   000
```

EXPLANATION. Three figures are required to contain the divisor, and in these figures, (153,) the divisor is contained 5 times. 5 is therefore the first quotient figure. Subtracting 5 times 29 from the divided figures, a remainder of 8 is obtained, to which the 4 in the dividend is brought down producing the number 84. This I divide as before and obtain the quotient figure 2; then multiply and subtract and find a remainder of 26. I then bring down the 1 and divide, obtaining the third figure of the quotient, viz., 9, and as all the figures are brought down and divided, 529 is the quotient required, and there is no remainder. Hence we learn that 29 and 529 are the factors of 15341. If either of these numbers be written down as many times as there are units in the other, *their sum will be the dividend.*

Division may therefore be proved by addition. In obtaining the proof by addition, if there be a remainder, it must also be added in with the other numbers.

Addition is not however the *most direct mode* of proving division. The quotient and divisor are the factors of the dividend, (§ **64**, 1.) Hence, the dividend is produced by the multiplication of these two factors; that is, we prove division most directly, *by multiplying together the divisor and quotient.* If there *be no remainder* left in performing the division, *the product will equal the dividend;* if there be a remainder, *that must be added* to the product of the divisor and quotient, to produce the dividend.

If the work of the following sum be carefully observed,

How may we prove division? If there be a remainder what is done? How best prove addition? What will the product be? If there was a remainder, what must be done?

it will be seen that each quotient figure is obtained by four successive steps following each other constantly in the same order. We first *divide;* second, *multiply;* third, *subtract;* and fourth, *bring down.*

2. Divide 6283488 by 58.

PERFORMED.

```
58)6283488(108336
   58
   ---
    483
    464
    ---
     194
     174
     ---
      208
      174
      ---
       348
       348
       ---
       000
```

PROOF.

```
 108336
     58
 ------
 866688
5416800
-------
6283488=div.
```

3. Divide 7461300 by 95 *Ans.* 78540.
4. Divide 946656 by 1038. *Ans.* 912.
5. Divide 255678 by 39. *Ans.* 6555, and 33 rem.
6. Divide 1299137 by 17. *Ans.* 76419, and 14 rem.
7. Divide 4712394981 by 489. *Ans.* 9636799, and 270 rem.
8. Divide 2686211248 by 296. *Ans.* 9075038.
9. Divide 5973467 by 243. *Ans.* 24582, and 41 rem.
10. Divide 69372168 by 342. *Ans.* 202842, and 204 rem.
11. Divide 1748 by 18. *Ans.* 97, and 2 rem.
12. Divide 1193288 by 45. *Ans.* 26517, and 23 rem.
13. Divide 47429 by 23. *Ans.* 2062, and 3 rem.
14. Divide 463582 by 39. *Ans.* 11886, and 28 rem.

15. If 246 men raise a fund of $175152, what is the average sum paid by each? *Ans.* $712.

16. If an estate of $65466 be divided equally among 6 heirs, what is the share of each? *Ans.* $10911.

17. If a bridge owned by 12 men afford an annual income of $2352, what is each man's share? *Ans.* $196.

How many steps and in what order? What are the steps?

18. If a man travel 36500 miles in one year, how many miles does he average daily? *Ans.* 100 miles.

19. If a horse run 378 miles in 42 hours, how many miles does he run per hour? *Ans.* 9 miles.

20. Light comes from the sun to the earth, a distance of 95000000 miles, in 8 minutes of time. How far is that per minute? *Ans.* 11875000 miles.

21. Suppose an apple-tree having 19 branches have 2964 apples on all its branches, how many does it average per branch? *Ans.* 156 apples.

22. If there are 2142 letters on a page and these letters are equally divided into 42 lines, how many letters are there in a line? *Ans.* 51 letters.

§ **82.** To divide by 10, 100, 1000, 10000, &c.

We place on the right of any number, which we wish to multiply by 10, 100, &c., as many ciphers as there are in that multiplier. (§ **67**.) Division being an operation directly the reverse of multiplication, to divide by these same numbers, viz., 10, 100, &c., *we cut off from the right of the dividend as many figures as there are ciphers in the divisor.* The figures on the *left of the dividend point*, constitute the *quotient*, and those on the *right*, the *remainder*.

1. Divide 76935 by 100. *Ans.* 769.35; that is, the quotient=769, the remainder=35.
2. Divide 8934678 by 10. *Ans.* 893467, and 8 rem.
3. Divide 3987069 by 1000. *Ans.* 3987.069.
4. Divide 18764307 by 10000. *Ans.* 1876.4307.
5. Divide 57306946 by 100. *Ans.* 573069.46.
6. Divide 4680135 by 1000. *Ans.* 4680.135.
7. Divide 778899664 by 10000. *Ans.* 77889.9664.

§ **83.** To divide by *any number, having one or more of its right-hand figures, ciphers.*

Cut off from the right of the divisor the ciphers, and

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82. How divide by 10, 100, &c.? Which figures constitute the quotient? Which the remainder? In 3d sum, (§ 82,) which figures form the quotient, and which the remainder?

83. How divide by any number having ciphers on its right? What is to be done with the figures cut off, if there be a remainder? What do they constitute, if no remainder be left after division?

from the right of the dividend, figures equal in number to those ciphers, and divide the remaining figures of the dividend by the significant figures of the divisor. If there be a remainder, annex to it the figures cut off from the dividend, to obtain the full remainder. If there be no remainder, the figures cut off will constitute the remainder.

1. Divide 8963757 by 3700.

PERFORMED. 37|00)89637|57(2422=quotient.

74

156

148

83

74

97

74

2357, total remainder.

2. Divide 876437 by 16900. *Ans.* 51, and 14537 rem.
3. Divide 6983465 by 89000. *Ans.* 78, and 41465 rem.
4. Divide 77883567 by 670. *Ans.* 116244, and 87 rem.
5. Divide 9567839706 by 378500. *Ans.* 25278, and 116706 rem.
6. Divide 3469786 by 860. *Ans.* 4034, and 546 rem.
7. Divide 398756946 by 5600. *Ans.* 71206, and 3346 rem.
8. Divide 76395784 by 63000. *Ans.* 1212, and 39784 rem.

APPLICATION OF PRECEDING RULES.

§ 84. Observe carefully the following four directions, viz.:

1st. To ascertain the total value of several articles of *unequal* value, *add their individual value.*

2d. To ascertain the total value of several articles of *uniform* value, *multiply that value by the number of articles.*

3d. When the value of two objects or sets of objects is

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84. How find the total value of several objects of dissimilar individual value? How find the total value of several articles of uniform value?

given, and the difference of their value required, *find that difference by subtraction.*

4th. To find the *individual* value of each of several objects of *uniform* value, their *total* value being given, or to find one of the equal portions into which any number is separated, *divide the total value or number by the specified number of objects.*

1. A having $7968, lends to B, $2369; to C, $1876; and to D, $2196. How many dollars has he left? (§ 54.)
Ans. $1527.

2. Bought a horse for $117, a yoke of oxen for $98, a carriage for $196, and an ox-cart for $49. How much did I pay for the whole? (§ 84, 1st.) *Ans.* $460.

3. Bought 239 yards of broadcloth at $7.50 a yard. How much did my purchase amount to? (§ 84, 2d.)
Ans. $1792.50.

4. A man sold two houses: one house for $7968, the other for $8690. He also bought two farms: one for $4896, and the other for $6893. How much money had he left? (§ 55; also 84, 3d.) *Ans.* $4869.

5. A merchant bought 675 yards of cloth for $4050. What was the average price per yard? (§ 84, 4th.)
Ans. $6.

6. A man bought a farm of 645 acres for $29025. How much did he pay per acre? (§ 84, 4th.) *Ans.* $45.

7. Suppose a farmer raises 67 bushels of wheat, valued at $1.25 per bushel; 118 bushels of corn, valued at 67 cents a bushel; and 279 bushels of oats, valued at 33 cents a bushel. What is the total value of his products? (§ 84, 2d and 1st: combine the two operations.) *Ans.* $254.88.

8. A man traveled 869 miles by stage, for which he paid at the rate of 5 cents a mile; 728 miles by railroad, which cost him at the rate of 3 cents a mile; and 963 miles by steamboat, at the rate of 4 cents a mile. How much money did he pay for stage, railroad, and steamboat fare? (§ 84, 2d and 1st.) *Ans.* $103.81.

How find the difference in the value of two objects or sets of objects? The total value of several objects of uniform value being *given, how find the* value of one?

9. A collector received from A, $87.92; from B, three times the sum A paid; and from C, twice the sum B paid. How much money did he receive from the three? (§ 84, 4th and 1st.) *Ans.* $879.20.

10. A farmer sold a sheep for $3.50; a cow for five times the price of the sheep; a horse for three times the price of the cow; and a yoke of oxen for twice the price of the horse. How much did he receive for the whole? *Ans.* $178.50.

11. A man's estate, which by will was to be divided equally among his six sons, consisted of $18970 of real estate, and $12788 of personal property. What was each son's share? (§ 84, 1st and 4th.) *Ans.* $5293.

12. How many times must $5293 be repeated to amount to the sum of $31758? *Ans.* 6 times.

13. From A, I collected $69; from B, $137; from C, $278; and from D, $636. I afterward paid away the whole sum collected for land, at $8 an acre. How many acres did I buy? (§ 84, 1st and 4th.) *Ans.* 140 acres.

14. A merchant bought 37 barrels of flour for $185; 378 barrels for $1512; and 1692 barrels for $8838. What was the *average* price per barrel? *Ans.* $5.

15. A man bought a farm for $8780, and sold it again at an advance of $2340. He then divided the money he received for it equally among his five sons. What did each son receive? (§ 84, 1st and 4th.) *Ans.* $2224.

16. A farmer sold his farm for $8972; his live-stock for $1869; his hay for $976; and his grain for $756. He then paid debts to the amount of $2773, and divided the remainder equally among his eight children. How much did each child receive? (§ 84, 1st, 3d, and 4th.) *Ans.* $1225.

17. An importer owning 8 vessels, valued at an average of $18963 per vessel, directed by his will that the total value of these 8 vessels should be equally divided among his 7 sons. What did each son receive? (§ 84, 2d and 4th.) *Ans.* $21672.

18. An importer of cloth imported of one quality of broadcloth, 780 yards; of a second quality, 540 yards; and of a third, 480 yards: the first at $5, the second at $4, and the *third at* $3 per yard. He manufactured the whole into *overcoats, each* coat requiring 4 yards of cloth, which he

sold at $20 each. What was the value of all the cloth he imported, how many coats did he make, and how much did he gain? (§ 84, 1st, 3d, and 4th.)

Ans. Value of cloth, $7500; coats, 450; gained $1500.

19. I bought 375 barrels of flour for $1875; 632 barrels for $3160; and 786 barrels for $3930. How many barrels of flour did I buy, how much money did I pay for the whole, and what is the average price per barrel?

Ans. 1793 bbls.; total cost, $8965; price per bbl., $5.

20. A farmer sold 5 horses at an average price of $75 each, and 6 yoke of oxen at $95 a yoke. To the sum thus received he added $55 from his pocket, and expended the whole for land at $25 an acre. How many acres did he buy? *Ans.* 40 acres.

21. A potato-field contains 1560 hills. If there be an average of 21 potatoes in a hill, and if 126 potatoes make a bushel, how many bushels of potatoes are there in the field?

Ans. 260 bushels.

22. Six men bought land in company, three of whom paid $972 each, and the remaining three $729 each. How much land did they buy, the average price being $7 per acre? *Ans.* 729 acres.

23. Suppose an estate to be worth $28560, against which there are claims to the amount of $8100. Now if, after deducting the claims, the remainder be equally divided among 5 individuals, what is each one's share? *Ans.* $4092.

24. If 8 bushels of apples make a barrel of cider, how many barrels can be made from the three following orchards, viz.: one of 55 trees, averaging 9 bushels of apples to each tree; a second of 63 trees, averaging 7 bushels to a tree; and a third of 72 trees, averaging 6 bushels to a tree?

Ans. 171 barrels.

25. Received from A, $756; from B, $1239; and from C, $854. From the amount thus received I paid $1400 for 350 yards of broadcloth, and expended the balance for molasses at $7 per barrel. How many barrels did I buy?

Ans. 207 barrels.

DECIMAL NUMBERS.

§ 85. The scholar has already been shown that simple numbers, commencing with the unit or right-hand figure and advancing towards the left-hand figure, increase in the constant and uniform ratio of 10. That is, all figures expressing numbers less than 10, occupy the right-hand (units) place; all figures expressing any number of tens, occupy the second place, or the place next on the left of units; all figures expressing hundreds, occupy the third place, or the place next on the left of tens, &c. (§ **2**, **4**, **7**, **9**, **10**, **11**, **12**, &c.)

Since then these figures increase uniformly in a ten-fold ratio from the right to the left, if we *reverse the order*, that is, if we enumerate their local value from the *left to the right*, their value will *decrease* in the same (tenfold) ratio from any denomination however high, to that of units.

If from the unit figure, we continue to advance towards the right, preserving the same ratio of decrease, we obtain numerical expressions of *less value than units or single whole objects*, decreasing in value in the constant ratio of 10. These are *tenths*, *hundredths*, *thousandths*, *tenths of thousandths*, &c., that is, *the whole object* is first *divided into* 10 *equal parts*, then *each of these parts is divided into* 10 *other equal parts*, each of 1 tenth the value of the preceding parts, and making 100 equal parts of the whole or integral object: *and again these latter parts are divided in the same manner*, making 1000 equal parts of the whole, and so on.

These parts of integers thus obtained, are called *decimals*, and although they express but *parts of numbers*, are capa-

85. How do simple numbers increase? When figures express numbers less than 10, what place do they occupy? Those expressing tens? All figures expressing hundreds? If we reverse the order of reading figures, will the local value increase or decrease? If we advance from the place of units, towards the right hand, what do we obtain? In what ratio do they decrease? What are those numbers called? How is the whole object divided? How then? &c. What *are these parts of* integers called?

ble of being *joined in the same expression with whole numbers*, since they increase and decrease *in the same ratio*. They may be read as follows, commencing with thousandths, viz., ten thousandths make one hundredth, ten hundredths make one tenth, ten tenths make one unit, ten units make one ten. The general principle is the same when we commence with any other denomination than thousandths.

The following table will illustrate the manner of writing integers and decimals in one number. The decimals are always written on the right of the whole numbers, with a period, called the decimal point, placed between them.

Hundreds of Thous.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	.	Tenths.	Hundredths.	Thousandths.	Tenths of Thousandths.	Hundredths of Thous.
3	4	5	6	7	8	.	1	5	7	4	6

If any figure stand alone, and has no decimal point on the left of it, it always expresses *units;* if a decimal point be placed on its left, it then expresses *tenths;* if two figures are written on the right of the decimal point, they together express hundredths; if three figures are thus written, they express thousandths, &c. Hence .5 is read *five tenths;* .50 is read fifty hundredths, and .500 is read *five hundred thousandths;* so also .56 is read *fifty-six hundredths;* .578, *five hundred and seventy-eight thousandths.*

It must be remembered that *units* are the lowest denomination *that expresses whole numbers;* that below that denomi-

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Express what? Can they be written with whole numbers? And why? Show the manner of reading decimals? How are decimals always written? What stands between the whole number and decimal? If a figure stand alone without a decimal point, what does it express? If a decimal point be placed on its left, what? If two figures are so written? If three? How is .5 read? How .50? .500? .56? .578?

nation *parts of units* only are expressed; and these, decreasing in the uniform ratio of 10, are distinguishable from whole numbers by being placed on the right of the period or decimal point. Thus, 78.598 is composed of the whole number 78 and the decimal .598.

It will be observed that when a whole object is divided into 10 equal parts, the ten parts will equal the whole one divided; and *to express only a part* of that object by a decimal, a decimal number less than 10, and composed of only one figure, must be employed. If the whole object be divided into 100 equal parts, any *assignable part* of that object can be expressed only by a decimal number less than 100, and consisting of *two* figures; and if it be divided into 1000 equal parts, any portion of the same is expressed by a number less than a thousand, and consisting of *three* figures. Hence, if a decimal consist of *one* figure only, it is *tenths;* if of *two* figures, it is *hundredths;* if of *three* figures, it is *thousandths;* and if of *four* figures, it is *tenths of thousandths*, &c. The scholar will read the following decimals, viz.: .7, .89, .176, .349, .67, .5. Hence, also, if any number of hundredths less than 10 are to be expressed, *a cipher* must be placed on the *left* of the figure expressing that number. Read .06, .09, .04, .08, &c. Also, if thousandths less than 10 are to be expressed, *two ciphers* must be placed on the *left* of the figure expressing the thousandths; and if less than 100, one cipher must be so placed. Read .007, .005, .004, .003, .009, &c.; also .076, .039, .057. If, then, a cipher be placed on the right of a figure expressing *tenths*, it becomes *hundredths;* if two ciphers are so placed,

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What is the lowest denomination expressing whole numbers? What next to units is expressed? In what ratio decreasing? How are these distinguished from whole numbers? Give an illustration? If a whole object is divided into 10 parts, how is a part of that object expressed? If the whole be divided into 100 equal parts, how can any assignable part of the same be expressed? How, if the whole be divided into 1000 equal parts? If a decimal consist of one figure, what is it? What, if it consist of two figures? If of three figures, &c.? How express a number of hundredths less than 10? If thousandths less than 10 are to be expressed, how is the decimal written? If a cipher be placed on the right of a decimal expressing tenths, *what does it become?*

it becomes *thousandths*, the value of each expression being the same. Hence ciphers placed on the right of a decimal increase the *number of parts only* in a tenfold ratio, while the *value* of the whole remains unaffected.

When whole and decimal numbers are written together, the numbers produced thereby are called *mixed numbers*.

The following are mixed numbers, and are thus read, viz.:

7.87	is read	7 and 87	hundredths.
89.7	"	89 and 7	tenths.
1.679	"	1 and 679	thousandths.
39.873	"	39 and 873	thousandths.
57.6345	"	57 and 6345	tenths of thousandths.

It should be remembered, that in reading both whole numbers and decimals, we must always commence the enumeration of the number with the *unit* figure. Commencing with units and advancing towards the left, we find the integral designations to be, units, tens, hundreds, thousands, &c.; if we advance towards the right, commencing with the same unit figure, they are, *units, tenths, hundredths, thousandths*, &c. Observe the differences in the reading.

Read the following numbers, &c.;

36.8; 5.78; 13.67; 4.156; 7.067; 0.4036; 3.3967; 81.456; 6.0007; 7.0109; 8.1001; 17.61091; 18.000006; 3.109763; 5.678; 116.2876; 108.303; 12.06; 13.007; 14.0003; 19.00006.

Write the following numbers in figures, viz.:

Seventy-eight and six tenths: written 78.6. Twenty-six and fifteen hundredths: written 26.15. In like manner, write ninety-eight and seventy-three hundredths. Nineteen and three hundred and fifty-nine thousandths. One thousand five hundred and sixty-nine and five tenths. Three hundred and eighty-seven, and four hundred and sixteen thousandths.

What if two ciphers are so placed? How the value? What is the effect of ciphers placed on the right of a decimal? When whole and decimal numbers are written together, what are they called? With what figure do we always commence the enumeration? What designations as we advance to the left? What, advancing to the right?

Write down sixteen and ninety-five hundredths.

Ans. 16.95.

Write down eighty-two and eight hundredths. *Ans.* 82.08. (§ 85.)

Write down ten and three hundredths; also, six and nine hundredths; two hundred and fifteen and seventeen thousandths; ninety-seven and one hundred and sixty-three thousandths.

ADDITION OF DECIMALS.

§ 86.—To add integral and decimal numbers, we have the following rule:

RULE.

Write the several numbers to be added, so that units shall stand under units, tens under tens, &c.; also, tenths under tenths, hundredths under hundredths, &c.

Commence with the lowest (right-hand) *denomination, and add, as in Simple Addition,* (§ 32,) *and from the right hand of the amount, cut off decimals equal in number to the greatest number of decimals in the given numbers.*

The decimal point in the amount will always *stand directly below the points in the given numbers.*

Add the following sums:

(1.)	(2.)	(3.)
8.76	15.879	27.18
19.56	8.007	396.109
163.4	447.056	1.679
378.59	36.606	281.456
81.05	18.518	303.003
651.36 amount.	526.066	1009.427

4. What is the amount of the following numbers, viz.: 3.19; 7.81; 116.932; 871.9; and 11.467? *Ans.* 1011.299.

86. How add integers and decimals? How will the points always stand?

5. What is the amount of 163.136; .97; 401.004; 5.019; 2.77; and 29.029? *Ans.* 601.928.

6. What is the amount of 1.006; 600.1; 89.19; 136.69; 30.8; 46.09; and 17.001? *Ans.* 920.877.

7. What is the amount of 87.166; 27.093; 88.099; 4.009; 17.076; and 186.38? *Ans.* 409.823.

8. What is the amount of 37.196; 88.996; 1.09; 93.059; 71.673; 96.837; 14.14; and 1.1? *Ans.* 404.091.

9. What is the amount of 5, and 9 tenths; 63, and 25 hundredths; 46, and 568 thousandths; 17, and 17 thousandths; and 9, and 1 thousandth? *Ans.* 141.736.

10. What is the amount of 18, and 7 hundredths; 9, and 56 thousandths; 189, and 9 thousandths; and 906, and 607 thousandths? *Ans.* 1122.742.

11. What is the amount of 867, and 49 thousandths; 17, and 867 thousandths; 216, and 9 tenths; 11, and 5 hundredths; and 3, and 6 tenths? *Ans.* 1116.466.

SUBTRACTION OF DECIMALS.

RULE.

§ 87. *Write the less number beneath the greater, placing units under units, tens under tens; tenths under tenths, hundredths under hundredths, &c.; and subtracting as in whole numbers, point off the decimals in the remainder as in the amount of the numbers given in Addition of Decimals.*

If in the subtrahend or lower number there are more decimals than in the minuend or upper number, the deficiency in the minuend may be made up with ciphers.

1. From 89.116 take 62.463.

PERFORMED.

89.116
62.463
26.653

Each number should be read by the scholar.

2. From 16.35 take 8.09. *Ans.* 8.26.

87. How subtract decimals?

3. From 19.189 take 12, and 9 tenths. *Ans.* 6.289.
4. From 20.7 take 18.99. *Ans.* 1.71.
5. From 89.763 take 40.11. *Ans.* 49.653.
6. From 87.367 take 8.8. *Ans.* 78.567.
7. From 18, and 69 hundredths, take 17, and 81 hundredths. *Ans.* .88.
8. From 186, and 091 thousandths, take 168, and 9 tenths. *Ans.* 17.191.
9. From 879, and 1 tenth, take 99, and 99 hundredths. *Ans.* 779.11.
10. From 1, and 899 thousandths, take 901 thousandths. *Ans.* .998.
11. From 23, and 679 thousandths, take 6, and 9 thousandths. *Ans.* 17.67.
12. From 19, and 87 hundredths, take 18, and 999 thousandths. *Ans.* .871.
13. From 58, and 15 thousandths, take 20, and 9 thousandths. *Ans.* 38.006.
14. From 87, and 9 tenths, take 78, and 9 hundredths. *Ans.* 9.81.

MULTIPLICATION OF DECIMALS.

RULE.

§ 88. *Multiply as in whole numbers, and point off from the product as many decimals as are equal to the decimals in both the multiplier and multiplicand. If the figures of the product are not equal in number to the decimals contained in both factors, supply the deficiency by prefixing ciphers.*

1. Multiply 36.19 by 8.27.

PERFORMED.

36.19
8.27
253.33
723.8
28952
299.2913

88. How multiply decimals?

Observe there are two decimals in the multiplier and multiplicand each; hence we cut off four decimals in the product.

2. Multiply 8.756 by 9.32. *Ans.* 81.60592.
3. Multiply 1.7156 by 2.913. *Ans.* 4.9975428.
4. Multiply 33.65 by 19.756. *Ans.* 664.78940.
5. Multiply 41.99 by 1.365. *Ans.* 57.31635.
6. Multiply 6.009 by 3.07. *Ans.* 18.44763.
7. Multiply 8.123 by .09. *Ans.* .73107.
8. If a man travel 3.5 miles in 1 hour, how far will he travel in 8.75 hours? *Ans.* 30.625 miles.
9. If a cistern hold 83.63 gallons, how many gallons will 8.25 such cisterns hold? *Ans.* 689.9475 gallons.
10. If one sack contain 876.93 pounds, how many pounds in 83 such sacks? *Ans.* 72785.19 pounds.
11. If a man spend .87 of a dollar in one day, how many dollars will he spend in 12.75 days? *Ans.* 11.0925 dollars.
12. If 1 barrel of molasses cost 5.25 dollars, how many dollars will 8.37 barrels cost? *Ans.* 43.9425 dollars.
13. If a load of wood cost 8.25 dollars, how much will 16.5 loads cost? *Ans.* 136.125 dollars.
14. One rod contains 16.5 feet, how many feet are there in 32.8 rods? *Ans.* 541.20 feet.

DIVISION OF DECIMALS.

RULE.

§ **89.** *Divide as in simple numbers,* (§ **79**, **81**,) *and point of from the right of the quotient figures, as many decimals as are equal to the excess of the decimals in the dividend over those in the divisor.*

Division is the reverse of multiplication; that is, the dividend is the product of two factors, one of which (the divisor) is given, and the other (the quotient) required. The dividend must then contain as many decimals as the divisor

89. What is the dividend? Which of the factors is given? How many decimals must the dividend contain?

and quotient together. (§ 88.) But the divisor being given and its decimal known, the excess of its decimals over those of the divisor, *must belong to the quotient*, the other factor of the dividend.

If the decimals in the divisor are equal to those of the dividend, and if there be no remainder after division, *the quotient is a whole number.* If there be a remainder, ciphers must be annexed, the division continued, and each cipher annexed, regarded as a decimal belonging to the dividend.

If in any instance the decimals of the divisor be more than those of the dividend, make them equal *by annexing ciphers.*

Whenever the figures of the quotient do not equal the excess of decimal places in the dividend over those in the divisor, make them equal by *prefixing ciphers* to the quotient.

1. Divide 34.317 by 21.75.

PERFORMED.

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21.75)34.317(1.577
      2175
     12567
     10875
      16920
      15225
       16950
       15225
        1725=rem.
```

Two ciphers have been added to the remainders of the dividend, making the whole number of decimals belonging to the dividend, five, and there are two in the divisor. Hence we point off three in the quotient.

What portion of the decimals of the dividend must belong to the divisor? If the decimals of the divisor and dividend be equal, and there be no remainder, what is the quotient? What must be done if there be a remainder? The decimals must equal what? If there be more decimals in the divisor than dividend, what is to be done? If, in any instance, the figures in the quotient are not equal to the excess of decimals in the dividend over those of the divisor, what must be done?

2. Divide 483.125 by 386.5. *Ans.* 1.25.
3. Divide 207.6032 by 8.96. *Ans.* 23.17.
4. Divide 99.99 by 33.3. *Ans.* 3.0027+.
5. Divide 65.8952 by 18.275. *Ans.* 3.605+.
6. Divide 1.9404 by 1.47. *Ans.* 1.32.

In dividing, if a remainder be finally left, the quotient is marked +, implying more. (See 4th sum.)

§ **90.**—To divide a decimal by 10, 100, 1000, &c.

RULE.

Remove the decimal point as many places to the left as there are ciphers in the divisor.

1. Divide 30515.50 by 100. *Ans.* 305.1550.
2. Divide 36.5 by 10. *Ans.* 3.65.
3. Divide 36.10 by 100. *Ans.* .365.
4. Divide 981 by 1000. *Ans.* .981.

5. If 36.34 bushels of corn grow on one acre, how many acres will produce 674 bushels? *Ans.* 18.547+ acres.
6. If 6 yards of cloth cost 24.48 dollars, what was the price of one yard? *Ans.* 4.08 dollars.
7. Bought 56.87 yards for 131.3697 dollars; what was the price of 1 yard? *Ans.* 2.31 dollars.
8. If 9.6 yards cost 61.632 dollars, what was the value of a yard? *Ans.* 6.42 dollars.
9. If 7.5 tons of hay cost 112.5 dollars, what is the price of a ton? *Ans.* 15 dollars.
10. Bought 17.5 acres of land for 148.75 dollars; what was the price of one acre? *Ans.* 8.50 dollars.
11. Bought 15.5 pounds of beef for 1.24 dollars; what was the price of a single pound? *Ans.* .08 dollar.
12. Bought 72.4 yards of tape for 2.172 dollars; what was the price of a yard? *Ans.* .03 dollar.

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90. How divide by 10, 100, 1000, &c.

FEDERAL MONEY.

§ **91.**—Federal money is the currency of the United States.

Its denominations are, mills, cents, dimes, dollars, and eagles. These are the regular denominations, and, like simple numbers, increase in a tenfold ratio. The following table gives the ratio and relative value of these denominations:

TABLE OF FEDERAL MONEY.

10 mills	(marked m.) make	1 cent,	marked	ct.
10 cents	"	1 dime,	"	d.
10 dimes	"	1 dollar,	"	$ or dol.
10 dollars	"	1 eagle,	"	E.

These are the regular coins of the United States. They *increase in a tenfold ratio.* The other coins, such as the 25 cent piece, the quarter, half, and double eagle, are only parts or repetitions of these. These coins are of three kinds, differing from each other as to their value and the metal from which they are manufactured.

The least valuable are the cent and half cent, which are composed of copper. Mills have never been coined, and are merely nominal. The next in value are the silver coins, viz.: the 5 cent piece, or half dime; the 10 cent piece, or dime; the 25 cent piece, or quarter of the dollar; the half dollar; and the dollar. The most valuable are the gold coins. These are the gold dollar, the quarter eagle, the half eagle, the eagle, and the double eagle. These are the only coins authorized by our government. Their values are:

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91. What is Federal money? What are its denominations? How increase? Recite the table. What coins not in the table? What are they? How many kinds of coin? How differing? What are the least valuable coins? Composed of what? Are mills coined? What are the silver coins? What are the most valuable coins? Name *them?*

THE GOLD COINS.		THE SILVER COINS.		THE COPPER COINS.
The double eagle,	$20.	The dollar,	100 cts.	The cent, or
The eagle,	$10.	The half dollar,	50 "	one hundredth
The half eagle,	$5.	The quarter dollar,	25 "	part of the dol-
The quarter eagle,	$2.50	The dime,	10 "	lar,—and the
The gold dollar,	$1.	The half dime,	5 "	half cent.

The $6\frac{1}{4}$ and $12\frac{1}{2}$ cent pieces, are not American coins; neither are a large portion of the pieces that pass for the quarter of a dollar. The American quarters are stamped with the American eagle.

The gold and silver coins are not composed of pure metal, but are alloys; that is, these metals are compounded with the baser metals. The purity of a metal is expressed by the word *carat:* a word indicative of a twenty-fourth part of a metal. For instance, if a metal is said to be 21 carats fine, the meaning is, that 21 twenty-fourth parts are pure metal, and 3 twenty-fourth parts are of a less valuable metal.

The alloy for gold is a mixture of silver and copper; the alloy for silver is pure copper.

The denominations of Federal Money increase and decrease *in precisely the same ratio as simple numbers and decimals;* that is, in a tenfold ratio. They are consequently added, subtracted, multiplied, and divided by the same rules.

The dollar is regarded as the unit, or *integral* object in Federal money, and is, therefore, the point at which numeration commences.

As we advance from this point towards the left, the denominations increase *constantly in the ratio of ten;* while those to the right of this point, decrease uniformly in the same ratio; that is, the *first* denomination on the right of

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Name all the gold coins—the silver coins—the copper coins. What are the $6\frac{1}{4}$ and $12\frac{1}{2}$ cents? Many quarter dollars? The American quarters, how stamped? Are gold and silver coins pure metal? What are they? What is an alloy? How express the purity of a metal? Meaning of the word carat? Illustrate? What is the alloy for gold?—for silver? The denominations of Federal money, how increase? How added, &c.? What is the unit of Federal money? Where does numeration commence? How increase, advancing to the left?

dollars, is *dimes* or tenths of dollars; the *second*, *cents* or hundredths of dollars; the *third*, *mills* or thousandths of dollars, &c.

Hence our computations in money *are all based on the decimal system*—a system increasing and decreasing through all its denominations in the *uniform ratio of* 10.

The dimes, cents, &c. of any number indicating Federal money, are marked by the period or decimal point, the same as in decimal numbers; that is, the first place on the right of the point is dimes; the second, cents; the third, mills, &c.; while on the left of that point, dollars stand first; eagles or tens of dollars, second; and so onward.

In reading any decimal number indicating Federal money, the terms, *dollars*, *cents*, and *mills*, are generally used; those of *eagles* and *dimes*, being seldom employed. Thus in reading $36.27, instead of saying, 3 eagles, 6 dollars, 2 dimes, and 7 cents, we say 36 dollars and 27 cents.

It will be remembered that Federal money is always designated by the character, $.

Read the following sums of Federal money, viz.: $3.78; $29.06; $303.639; $73.086; $136.136; $208.129; $33.333; $55.884; $72.198; $67.493; $128.388; $1962.481; $0.798; $13.318; $66.999; $83.38; $9.006; $1.108; $3.07; $860.46; $569.336; $28.963; $17.177.

Notice carefully the following facts. 1st. Cents are reduced to mills by annexing *one* cipher; thus 8 cents=80 mills. 2d. Dollars are reduced to cents by annexing *two* ciphers; thus $5=500 cents. Dollars are likewise reduced to mills by annexing *three* ciphers; thus $5=5000 mills. (§ **67** and **91**, Table.) 3d. Mills are reduced to cents by dividing by 10, that is, cutting off *one* figure from the right hand of

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How those to the right? What stands first on the right of dollars?—what second?—third? On what system are computations in money made? What peculiarity of that system? How are dimes and cents marked? Explain? What terms used in reading Federal money? Illustrate? By what character is Federal money designated? How are cents reduced to mills?—dollars to cents?—dollars to mills? How are mills reduced to cents?—mills to dollars?—cents to *dollars*?

the number expressing the mills; thus 5678 mills=567.8 cents. Mills are reduced to dollars by cutting off *three* right-hand figures from any number expressing mills; thus, 5678 mills=5.678 dollars: and finally, cents are reduced to dollars by cutting off *two* right-hand figures from any number expressing cents. (§ 82.)

1. Reduce 78 cents to mills. *Ans.* 780 mills.
2. Reduce 39 cents to mills. *Ans.* 390 mills.
3. Reduce 47 dollars to cents and mills.
Ans. 4700 cents, 47000 mills.
4. Reduce 8 dollars to cents and mills.
Ans. 800 cents, 8000 mills.
5. Reduce 790 mills to cents. *Ans.* 79 cents.
6. Reduce 8700 cents to dollars. *Ans.* $87.
7. Reduce 8769 cents to dollars. *Ans.* $87.69.
8. Reduce 7693 mills to cents, and to dollars.
Ans. 769.3 cents, $7.693.
9. Reduce $18 to cents and mills.
Ans. 1800 cents, 18000 mills.
10. Reduce $36.19 to mills. *Ans.* 36190 mills.
11. Find the amount in mills of $37.63; 96 cents; 9 mills; $6.73; 8 mills; and 98 cents.
Ans. 106317 mills.
12. Find the amount in mills of $1.009; 37 cents; $16.319; 116 mills; $5.123; 3 mills; and 87 cents.
Ans. 23810 mills.
13. Find the amount of 69 cents; $1.873; 758 mills; 3 mills; $7; 3 cents; and 9 mills. *Ans.* $10.363.
14. Find the amount of $7.39; $3.217; $6.198; $5.91; $22.17; and $109.01. *Ans.* $153.895.

ADDITION OF FEDERAL MONEY.

RULE.

§ 92. *Write the several numbers under each other, placing dollars under dollars, cents under cents, &c., and place the point as in addition of decimals.* (§ 86.)

1. What is the amount of $17.56; $93.125; $189.17; $256.188; $53.17; and $91.875?

PERFORMED.

17.56
93.125
189.17
256.188
53.17
91.875
701.088 amount.

It will be observed that the decimal points stand directly under each other, if the numbers are correctly written.

2. What is the amount of $1.55; $0.72; $340.89; $0.01; and $1460.99? *Ans.* $1804.16.

3. What is the amount of $17.843; $49.637; $1.01; $77.007; $116.39; $24.93; and $176.835?

Ans. $463.652.

4. What is the amount of $12.07; $13.70; $18.93; $27.086; $4.777; $3.99; $77.937; and $16.12?

Ans. $174.61.

5. What is the amount of $23.13; $268.076; $489.807; $33.666; $15.09; $81.186; $39.167; and $56.783?

Ans. $1006.905.

6. What is the amount of $39.93; $87.675; $56.428; $19.193; $12.273; $368.863; $15.586; and $87?

Ans. $686.948.

7. What is the amount of $17.174; $201.127; $13; $66.968; $87.678; $56.13; $69.47; and $0.169?

Ans. $511.716.

8. A farmer bought a cow for $23.75; a yoke of oxen for $96.78; a horse for $69.82; and a pig for $1.625. What was the whole cost? *Ans.* $191.975.

9. A grocer paid for a box of cheese, $17.21; for candles, $8.32; for a cask of wine, $7.38; and for a box of raisins, $3.625. The whole cost is required.

Ans. $36.535.

10. Paid for building my house, $2169.72; for my barn,

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92. How write the several numbers? Where place the point in the *amount*?

$972.87 for my out-houses, $1272.69; and for digging my well $56.38. Required my whole expense.

Ans. $4471.66.

SUBTRACTION OF FEDERAL MONEY.

RULE.

§ **93.** *Write the numbers as in simple subtraction, then subtract, and place the decimal point as in addition of Federal money.*

1. From $463.42 take $399.99. *Rem.* $63.43.
2. From $179.364 take $88.449. *Rem.* $90.915.
3. From $125 take $9.09. *Rem.* $115.91.
4. From $642.99 take $99.99. *Rem.* $543.
5. From $127.01 take $41.10. *Rem.* $85.91.
6. From $200 take $0.90. *Rem.* $199.10.
7. From $2 take $0.05. *Rem.* $1.95.
8. From $99 take $0.99. *Rem.* $98.01.

9. I have $473, and my brother twice that sum, lacking 90 cents. How many dollars has my brother?

Ans. $945.10.

10. Having in my possession $1500, I bought a span of horses for $516.95; a carriage for $156.65; a gold watch for $221.19; and spent in traveling, $450.72. How much had I left?

Ans. $154.49.

MULTIPLICATION OF FEDERAL MONEY.

RULE.

§ **94.** *Multiply as in decimal numbers.* (§ 88.)

Multiplication in Federal money is employed for ascertaining the value of any number of articles, the price of one article being given. It is obvious that the price of a single

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93. What is the rule for the subtraction of Federal money?

94. What is the rule for the multiplication of Federal money? For what is multiplication employed? How often must the price of a *single article* be repeated, to find the value of several articles?

article must be repeated as many times as there are articles, (whenever these articles are of uniform value,) to find their total value. The *number of articles* then determines the number of repetitions, and, of course, becomes the multiplier.

1. What cost 36 yards of cloth at $4.50 per yard?

Ans. $162.

PERFORMED.

4.50
36
2700
1350
$162.00 required cost.

2. What cost 29 pairs of shoes at $1.50 a pair?

Ans. $43.50.

3. What cost 35 pounds of beef at 8 cents a pound?

Ans. $2.80.

4. Bought 280 reams of paper, at $2.35 a ream; what was the whole cost? *Ans.* $658.00.

5. What cost 600 pounds of lard at 15 cents a pound?

Ans. $90.00.

6. If 1 ton of hay cost $16.42, what will 15 tons cost?

Ans. $246.30.

7. What is the cost of 349 acres of land, at $15.49 per acre? *Ans.* $5406.01.

8. Bought 18 yoke of oxen at $72.50 a yoke; what did they cost? *Ans.* $1305.00.

9. Bought 32 pounds of butter, at 20 cents a pound; 45 pounds of loaf-sugar, at 18 cents a pound; 56 pounds of coffee, at 15 cents a pound; 26 pounds of tea, at $1.75 per pound; 21 hundredweight of raisins, at $6.75 per hundredweight; 42 barrels of flour, at $7.50 a barrel; and 29 pairs of boots, at $4.50 a pair. What did the whole cost me?

Ans. $655.65.

What does the number of articles determine?

DIVISION OF FEDERAL MONEY.

§ 95. Division of Federal Money is employed to find the value of a single object, the price of several being given.

RULE.

Divide the total value of any given number of objects by that number of objects. The quotient will be the value of a single object.

The quotient always agrees in kind with the dividend, it being a part of the same, (§ **78**,) if that is cents or mills, the quotient is the same. Hence, in Division of *Federal money, we cut off as many decimals in the quotient, as there are in the dividend, including the ciphers* added to complete the divisions.

1. If 9 pounds of butter cost $2.25, what is the value of 1 pound? *Ans.* $0.25.

NOTE. Observe that $2.25 is to be separated into 9 *equal portions*, of which the quotient obtained by division is always *one*.

2. If 69 bushels of wheat cost $62.50, what is the cost of a single bushel? *Ans.* $0.905+.

3. If 500 pounds of butter cost $75, what is the price of 1 pound? *Ans.* $0.15.

4. If 15 tons of hay cost $311.70, what is the value of 1 ton? *Ans.* $20.78.

5. Paid $168.48 for 144 pounds of tea; required the price of one pound? *Ans.* $1.17.

6. Bought 50 firkins of butter for $375; how much did I pay a firkin? *Ans.* $7.50.

7. Paid $43.79 for 29 pairs of boots; what was the price of a single pair? *Ans.* $1.51.

8. If I pay $2.80 for 35 pounds of beef, what is the price of 1 pound? *Ans.* $0.08.

9. If an estate valued at $12000 be equally divided between 16 persons, how much will each receive? *Ans.* $750.

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95. When is division of Federal money employed? What is the rule? With what does the quotient agree in kind? How many decimals do we cut off in the quotient in the division of Federal *money?*

BILLS OF MERCHANDISE.

§ **96.** To compute the cost of a Bill of Merchandise, *find the value of each article named in the bill, by multiplying the price by the number of units in that article, and then add the value of the several articles.*

1:

J. Johnson, Esq.

Bought of Simpson & Co.

27 pounds of Sugar, at 9 cents a pound, - - -	$2.43.
18 pounds of Cheese, at 8 cents a pound, - - -	1.44.
30 pounds of Raisins, at 14 cents a pound, - -	4.20.
40 pounds of Coffee, at 12 cents a pound, - - -	4.80.
9 dozen Eggs, at 15 cents a dozen, - - - - -	1.35.
26 pounds of Butter, at 17 cents a pound, - - -	4.42.
What was the amount of the bill? *Ans.*	$18.64.

2.

T. Grundy

Bought of J. Sanford & Co.

28 yards of Silk, at $1.25 per yard, - - - -	$
38 yards of Muslin, at 36 cents per yard, - - -	
17 pairs of Cotton Hose, at 18 cents per pair, -	
26 skeins of Sewing Silk, at 5 cents a skein, - -	
16 yards of Cambric, at 21 cents per yard, - -	
Required the amount of the bill. *Ans.*	$56.40.

3.

G. Fanshaw, Esq.

Bought of Samuel Slocum,

63 barrels of Flour, at $4.50 per barrel, - - -	$
29 bushels of Corn, at 67 cents per bushel, - -	
73 bushels of Rye, at 48 cents per bushel, - -	
27 bushels of Barley, at 56 cents per bushel, -	
96 pounds of Lard, at 11 cents per pound, - -	
33 pounds of Salt Pork, at 8 cents per pound, -	
Required the amount of the bill. *Ans.*	$366.29.

96. How compute the cost of a bill of merchandise?

4.

J. B. Crawford

Bought of Sullivan & Co.

53 Morse's Geographies, at 56 cents, - - - - - $
75 Webster's Spellers, at 7 cents, - - - - - -
50 Ollendorf's French Readers, at 47 cents, - -
27 Day's Algebras, at 62 cents, - - - - - -

Required the amount of the bill. *Ans.* $85.47.

5.

Peter Prince, Esq.

Bought of S. Oliver,

8 Dress Coats, at $15.36 each, - - - - - - $
12 Frock Coats, at $16.20 each, - - - - - -
18 Velvet Vests, at $6.56 each, - - - - - -
21 Pairs of Pantaloons, at $5.75 a pair, - - -
16 Cloth Vests, at $4.63 each, - - - - - -
12 pairs of Drawers, at $2.12 per pair, - - -

Required the amount of the bill. *Ans.* $655.63.

6.

J. Jordan, Esq.

Bought of N. T. Sweezy & Co.

21 barrels Superfine Flour, at $6.37, - - - $
36 firkins Butter, at $6.30 a firkin, - - - -
19 kegs of Lard, at $3.20 a keg, - - - -
27 Beef Hams, at $2.67 each, - - - - -
86 pounds best Pork Ham, at 9 cents, - - -
117 pounds best Crushed Sugar, at 11 cents, -

Required the amount of the bill. *Ans.* $514.07.

7.

J. G. Bennett

Bought of Butler, King, & Co.

15 hogsheads Molasses, at $15.80 per hogshead, $
2800 pounds Brown Sugar, at 8 cents per pound, -
1200 pounds Coffee, at 13 cents per pound, - - -
3600 pounds Rice, at 3 cents per pound, - - - -
876 boxes Oranges, at $3.88 per box, - - - -
379 boxes Tea, at $32.18 per box, - - - - -

Required the amount of the bill. Ans. $16320.10.

8.

S. Wood, Esq.

Bought of Robert Baker & Co.

12 hogsheads Molasses, at $9.63, - - - - - $
36 hogsheads Brown Sugar, at $12.86, - - -
56 firkins Butter, at $6 a firkin, - - - - -
267 boxes Raisins, at $1.63 a box, - - - -
396 pounds Orange Co. Cheese, at 8 cents a pound,
87 pounds Java Coffee, at 13 cents per pound,

Required the amount of the bill. *Ans.* $1392.72.

9.

Robt. Lawton Esq.

Bought of Sylvester Summers,

87 yards of Broadcloth, at $2.21 per yard, - - $
96 yards Red Flannel, at 33 cents per yard, - -
83 yards Satinet, at 56 cents per yard, - - -
56 dozen Coat Buttons, at 37 cents per dozen, -
106 yards of Cotton Flannel, at 9 cents per yard,
53 yards best Silk Vesting, at $3.27 per yard, -

Required the amount of the bill. *Ans.* $474.00

10.

P. Paulding, Esq.

Bought of R. Richards,

73 Webster's School Dictionaries, at 42 cents, - - $
88 Webster's Spellers, at 7 cents, - - - - -
28 Worcester's Dictionaries, at $2.25, - - - -
93 Olney's School Geographies, with Atlas, at 87 cents, - - - - - - - - - - - - -
66 Sander's Readers, at 41 cents, - - - - -

Required the amount of the bill. *Ans.* $207.79.

11.

Joseph Thornton, Esq.

Bought of S. Severance,

18 tons Liverpool coal, at $6.50 per ton, - - - $
85 tons Lehigh coal, at $4.37 per ton, - - - -
63 tons Lackawanna coal, at $5.12 per ton, - -
48 tons Peach Orchard coal, at $5.30 per ton, -

Required the amount of the bill. Ans. $1065.41.

CANCELLATION.

§ **97.** We have, thus far, examined the four simple rules of arithmetic, viz.: Addition, Subtraction, Multiplication, and Division, as applied both to whole, decimal, and federal numbers.

In doing this, we have repeatedly seen that two or more of the simple operations are required to meet the conditions of a single sum, and are employed in the solution of the same arithmetical question. *Whenever this is the case, important abbreviations may often be effected.* If, for example, it becomes necessary to subtract 27 from any given number, and to add 35 to the remainder, the true result is obtained by adding the difference of these two numbers (35—27=8,) to that given number. In this case, we add the difference, because the *greater* of the two numbers is to be added. If the *greater* of two numbers is to be *subtracted*, and the less added, we *subtract* their difference.

Similar abbreviations are numerous and important in all operations in which *multiplication and division* are combined.

When any number is exactly produced by the multiplication of two or more smaller numbers, these smaller numbers are called the *factors* or *makers* of that number: thus 9 and 2, or 3 and 6, are the producers or makers of 18.

It often happens that *two or more numbers contain each the same factor:* thus, the numbers 12 and 16 contain each the factor 4; for $12=4\times3$, and $16=4\times4$.

Whenever *one* of any two numbers, *containing a common factor*, sustains to an arithmetical operation *the relation of multiplier*, and *the other*, the relation of *divisor*, that common factor may be rejected from each of the two numbers, and the remaining factors of the two numbers employed in their place. For example: if it be required to

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97. What, thus far examined? What have we seen? What may then be done? Illustrate? When are similar abbreviations important? What are the factors of a number? What often happens? When may a common factor be rejected? What employed instead? *Illustrate?*

multiply a certain number by 21, and divide the product by 14; from the 21 and 14 reject the common factor 7, and employ the remaining factors 2 and 3. The result is not affected by the rejection of that factor.

This rejection of a common factor is called *Cancellation.*

ILLUSTRATION. Suppose it be required to multiply 36 by 24, and divide the product by 18. First, we will arrange these numbers so that they may be conveniently compared with each other; we will *place those numbers, whose product forms a dividend, above a horizontal line, and the divisor below the same line;* thus, $\frac{36.\quad 24}{18}$.

Now it is obvious that 24 and 18 contain the common factor 6, which being rejected, the dissimilar factors 4 and 3 are left—the four above, and the 3 below the line, thus:

$$\frac{36\ \not{24}^{4}}{\not{18}_{3}}.$$

We may, therefore, multiply 36 by 4, and divide the product by 3 without altering the result. It is furthermore obvious, that the divisor 3, may itself be canceled in 36, and 12 obtained as a factor of the dividend; thus:

$$\frac{\not{36}^{12}\ \not{24}^{4}}{\not{18}_{\not{3}}}.$$

The factors below the line are now all canceled, while 12 and 4 are left uncanceled above the line. Their product constitutes the true quotient of $\overline{36 \times 24} \div 18$, viz., 48, for $12 \times 4 = 48$.

2. In like manner, multiply 26 by 36, and divide the product by 24, canceling the common factor 12. *Ans.* 39.

3. So also, 56 by 27, and divide the product by 36. *Ans.* 42.

4. Divide the product of 48 and 49 by 42. *Ans.* 56.

5. Divide the product of 75 and 81 by 45. *Ans.* 135.

6. Divide the product of 96 and 40 by 120. *Ans.* 32.

What is this called? Illustrate? When the factors below are all *canceled, what is* the true quotient?

7. Divide the product of 77 and 18 by 66. *Ans.* 21.
8. Divide the product of 56 and 45 by 72. *Ans.* 35.

By rejecting the common factors in the above examples, much labor is saved.

If the terms are more numerous than in the above examples, the work of rejecting factors may be continued, *so long as any factors can be found common to the terms forming the dividend and divisor.*

1. Multiply 36 by 27 and 15, and divide the product by 9, 6, and 10.

Statement: $\frac{36.\ 27.\ 15}{9.\ 6.\ 10}$.

$$\text{The same canceled: } \frac{\overset{3}{\cancel{36}}.\ \overset{3}{\cancel{27}}.\ \overset{3}{\cancel{15}}}{\underset{2}{\cancel{9}}.\ \cancel{6}.\ \cancel{10}}; \quad 3\times3\times3=27, \textit{ Ans.}$$

2. Multiply 81 by 64 and 16, and divide the product by 72 and 24. *Ans.* 48.
3. Divide the product of 132, 36, and 21, by 88 and 63. *Ans.* 18.
4. Divide the product of 63, 49, and 16, by 36 and 28. *Ans.* 49.

§ **98.** From the examples here given, it is obvious that any arithmetical operation performed by the combined application of multiplication and division, can be very expeditiously solved *by the rejection of all common factors.*

RULE.

Analyze the given sum so as to determine the multipliers and divisors requisite to obtain the required result. Then, placing the number to be operated upon and the required multipliers above a horizontal line, and the divisors below the same, reject equal factors from each; after which, complete the work by performing the necessary multiplication and division.

How long continue to reject factors?
98. What sums may be canceled? What is the rule?

1. A man hired himself to a farmer for 12 months at $15 per month, and expended the whole money, in the purchase of cows, at $18 each; how many cows did he purchase?

It is obvious that 15×12=the dollars received for his labor, viz., $180; $180\div18=10$, the cows purchased.

Canceled: $\frac{\overset{5.}{\cancel{15}}\ \overset{2}{\cancel{12}}}{\underset{\$}{\cancel{18}}}$; and $5\times2=10$. From 12 and 18 we reject the common factor 6; also, cancel the 3 now below the line in the 15 above the same.

2. A merchant exchanged 21 bushels of wheat valued at $2 a bushel, for broadcloth worth $6 per yard; how many yards did he buy? *Ans.* 7 yards.

3. A farmer gathered a field of corn containing 63 rows, of 28 hills each. Allowing 36 hills to yield a bushel, how many bushels were there in the field? *Ans.* 49.

4. A farmer hired 8 men for 6 months, at $12 a month each, and paid them in hay at $18 a ton; how many tons of hay were required? *Ans.* 32 tons.

5. A dealer in books bought 42 Webster's dictionaries at $4 each, and paid for the same in ledgers at $6 each; how many ledgers did it require? *Ans.* 28 ledgers.

6. A tailor made 81 military coats valued at $21 each, and received in pay flour at $9 a barrel; how many barrels of flour did it require to pay him? *Ans.* 189 barrels.

FRACTIONS.

§ 99. We have thus far spoken of numbers as representing whole objects, and also, as representing decimal parts of objects; that is, *parts decreasing in value in the constant ratio of ten.* We are now to regard a whole object as divided into *any number of equal parts.*

99. How thus far spoken of numbers? What now entered upon? What are fractions?

When a whole object is divided into any number of equal parts, other than 10, 100, 1000, &c., the arithmetical expressions representing those parts are called *Fractions*.

The *name* of a fraction is determined by *the number of equal parts into which the whole object is divided*, and the *value* of each of these parts will in all cases depend upon their *number*, that is, the *greater* the number the *less* the value of *each individual part*. If any single object be divided into *two* equal parts, each part is called *a half;* if into *three* equal parts, each part is called a *third;* if into *four* equal parts, each part is called a *fourth;* if into *five*, a *fifth;* if into *six*, a *sixth;* &c., the *number of parts* in every case giving name to the fraction.

Fractions always require *two numbers* or terms to express their value. One of these always stands *below* a short horizontal line, and indicates *the number of equal parts* into which any whole object is divided. *This is called the denominator;* thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, the 2, 3, and 4 are the denominators. The first one, 2, shows that the whole object is divided into *two* equal parts; the second one, 3, that it is divided into *three* equal parts; the third one, 4, that it is divided into *four* equal parts. The same is true of all denominators.

The other term, called the *numerator*, always stands *above* the same horizontal line; thus of the fractions, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{1}$, &c., the upper numbers, 1, 2, 3, and 4 are the numerators. This term always expresses *the number of those parts*, indicated by the denominator of a fraction *which constitute its value*. The greater the *denominator* of any fraction, therefore, the *greater the number of parts, but the less the value of each individual part;* while the greater the *numerator*, the *greater the value* of the fractions. Thus, in the fractions $\frac{1}{8}$, $\frac{2}{8}$, $\frac{5}{8}$, $\frac{7}{8}$. The *number* of parts into which the integer is divided by these fractions is 8. The first fraction expresses 1 of

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How determine the name of a fraction? Upon what will the value of each part depend? Illustrate. What gives name to every fraction? How many numbers required? How do they stand? What called, and showing what? Where stands the other term? What does the numerator express? What is said of the number of parts, and the value of each? How with the numerator? Illustrate.

these parts; the second, 2 of the parts, and consequently is twice the value of the first; the third, 5 of the same parts, and consequently equals five times the value of the first; while the last, $\frac{7}{8}$, obviously embraces the value of the whole object divided, wanting 1 eighth part only.

§ 100. Hence to obtain one *half*, *third*, or *fourth* part, &c., of any number, we divide that number *by the denominator of the fraction expressing the parts*, and to obtain any given number of such parts, we *multiply that part by the number of parts expressed by the numerator of the same fraction*.

1. Divide 12 into 3 equal parts. What is one of those parts, and what fraction of the whole number is it? What number is 2 third parts of 12, and what fraction of the whole number is it?

EXPLANATION. The number 12 equals three 4s, consequently 4 is 1 third of 12, and is expressed by the fraction, $\frac{1}{3}$, (one third,) while 2 third parts of 12 must be twice 1 third part, or $4 \times 2 = 8$, and is expressed by the fraction $\frac{2}{3}$.

2. What is 1 fourth of 8? 2 fourths? 3 fourths? How are 1 fourth, 2 fourths, &c., written?

3. What is 1 fifth of 20? 2 fifths of 20? 3 fifths of 20? 4 fifths of 20? 5 fifths of 20? How many *fifths* make a whole object or number?

4. What is 1 sixth of 12? 2 sixths of 12? 3 sixths? 4 sixths? 5 sixths?

5. What is 1 sixth of 24? 2 sixths of 24? 3 sixths? 4 sixths? 5 sixths?

6. What is 1 seventh of 35? 2 sevenths of 35? 3 sevenths? 4 sevenths? 5 sevenths? 6 sevenths? How many sevenths make a whole one?

7. What is 1 eighth of 16? 2 eighths of 16? 3 eighths? 4 eighths? 6 eighths? 7 eighths?

8. What is 1 ninth of 36? 2 ninths of 36? 6 ninths? 8 ninths? 7 ninths? 5 ninths?

9. What is 1 tenth of 40? 5 tenths of 40? 3 tenths? 7 tenths? 4 tenths? 9 tenths?

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100. How find one half, one third, &c., of a number? How obtain *any given number* of such parts?

§ **101.** It is often required to express by a fraction, what part any one number is of another number.

1. What part of 8 is 3? Since 8 contains eight units, and 3 contains 3 units, it is obvious that 3 is 3 eighths of 8, or $\frac{3}{8}$.

2. What part of 6 is 2? *Ans.* $\frac{2}{6}$ or 2 sixths of 6. What part of 6 is 3? 4? 5?

3. What part of 9 is 7? 5? 3? 4? 6? 2? 1? 8?

4. What part of 7 is 2? 6? 5? 3? 4? 1?

5. What part of 12 is 7? 3? 8? 9? 5? 4? 6? 11?

6. What part of 11 is 8? 7? 5? 1? 4? 3? 2? 10?

7. What part of 14 is 11? 9? 12? 8? 7? 5? 4? 13?

8. What part of 13 is 7? 8? 12? 6? 1? 3? 5? 8?

§ **102.** If the price of any one part of an object be given, *the price of the whole is found by multiplying that price by the number of such parts in the whole.* If however the price of a *number* of parts be given, *divide that price by the number of parts given, and multiply the quotient by the number of such parts as are equal to whole objects.* The product will be the price of the whole or integral object.

1. If $\frac{1}{2}$ pint of cherries cost 2 cents, what will a whole pint cost?

EXPLANATION. The whole of any object equals its 2 halves, and will consequently cost twice as much as its half. Hence a whole pint will cost two times 2 cents, and two times 2 cents is 4 cents, *Ans.*

2. If $\frac{1}{3}$ of a pound of beef cost 4 cents, what will a pound cost?

3. If $\frac{1}{4}$ of a ton of hay cost \$5, what will a whole ton cost?

4. If $\frac{3}{4}$ of a ton of hay cost \$18, what will a ton cost?

EXPLANATION. Since 3 fourths cost \$18, 1 fourth will

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101. What is often required? Illustrate.

102. When the price of one part of an object be given, how find the price of the whole? How if the price of a number of parts be given? Illustrate.

cost $\frac{1}{3}$ of $18, or $6; and if 1 fourth cost $6, 4 fourths, or the whole ton will cost 4 times $6, and 4 times $6 are $24.

Ans. $24.

5. If $\frac{5}{6}$ of a barrel of flour cost $10, what will a whole barrel cost?

6. If $\frac{7}{8}$ of a cord of wood cost $14, what will a cord cost?

7. If $\frac{3}{5}$ of a barrel of pork cost $9, what will a barrel cost?

8. If $\frac{3}{4}$ of a chest of tea cost $8, what will a whole chest cost?

9. If $\frac{1}{12}$ of a hogshead of wine cost $3, what will a whole hogshead cost?

10. If $\frac{6}{7}$ of a box of sugar cost $15, what will a whole box cost?

11. If $\frac{6}{7}$ of a ton of cheese cost $42, what will a ton cost?

12. If $\frac{1}{12}$ of an acre of land cost $44, what will a whole acre cost?

13. If $\frac{2}{11}$ of a bushel of corn be worth 8 cents, what is the value of a bushel?

14. If $\frac{3}{13}$ of a hogshead of rice cost $9, what will a whole hogshead cost?

15. If $\frac{3}{7}$ of a ton of iron cost $9, what will a ton cost?

16. If $\frac{5}{9}$ of a piece of land be valued at $45, what is the value of the whole?

§ **103.** When any given number is a fractional part of another number, that other number is found *by dividing the given number by the number of parts given, and multiplying the quotient by the parts required.*

1. Of what number is 9 the $\frac{3}{4}$ part?

EXPLANATION. If 9 is 3 fourths, 3 must be 1 fourth; and if 3 be 1 fourth, 4 times 1 fourth must be the required number. Hence, $3 \times 4 = 12$, *Ans.*

2. 8 is $\frac{2}{3}$ of what number? $\frac{4}{5}$ of what number?

3. 15 is $\frac{3}{5}$ of what number? $\frac{5}{8}$ of what number?

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103. When one number is a fractional part of another, how find the *other number?* Illustrate.

4. 18 is $\frac{6}{7}$ of what number? $\frac{3}{4}$ of what number?
5. 21 is $\frac{7}{8}$ of what number? $\frac{7}{4}$ of what number?
6. 24 is $\frac{3}{4}$ of what number? $\frac{6}{7}$ of what number?

§ **104.** Fractions always express *parts* of single objects, and are expressed by *two numbers*, written the one above and the other below a short horizontal line. The one *below* the line is called the *denominator*, showing the number of parts into which an object is divided; and that *above* the line is called the *numerator*, indicating the number of parts included in the fraction.

One half is written $\frac{1}{2}$; one third, $\frac{1}{3}$; one fourth, $\frac{1}{4}$; five eighths, $\frac{5}{8}$; seven ninths, $\frac{7}{9}$, &c.

§ **105.** There are several kinds of fractions, of which the following are the most important, viz.:

The *proper* fraction, the numerator of which is less than the denominator; as $\frac{4}{5}$, $\frac{2}{3}$, $\frac{3}{7}$, &c.

The *improper* fraction, the numerator of which is equal to, or larger than, the denominator; as $\frac{5}{4}$, $\frac{6}{5}$, $\frac{8}{8}$, &c.

The *simple* fraction, consisting of a *single* fraction, and may be either proper or improper; thus, $\frac{5}{6}$, $\frac{6}{5}$, $\frac{3}{8}$, &c.

The *compound* fraction, or fraction of a fraction; as $\frac{1}{3}$ of $\frac{5}{8}$, $\frac{3}{8}$ of $\frac{5}{6}$ of $\frac{2}{3}$, &c.

The mixed number, consisting of a whole number and a fraction written together; as $6\frac{2}{4}$, $5\frac{6}{7}$, &c.; and finally,

The *complex* fraction, or a fraction with a fraction in its numerator or denominator, or both; as $\frac{8\frac{3}{4}}{9}$, $\frac{6}{7\frac{1}{2}}$, $\frac{5\frac{1}{2}}{7\frac{3}{4}}$, &c.

§ **106.** The value of any fraction obviously depends on the *relative value of the numerator and denominator*; if the numerator be small compared with the denominator, the value will also be small. Thus, $\frac{3}{59}$ implies that an object is divided into 59 equal parts, and 3 of these parts are

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104. What do fractions always express? How expressed themselves? How written? The one below the line is called what? Shows what? The one above is called what? Shows what?

105. What is a proper fraction? An improper fraction? A simple fraction? A compound fraction? A mixed fraction? A complex fraction?

106. Upon what does the value of a fraction depend? Illustrate.

included in the fraction. $\frac{23}{24}$ expresses the division of an object into 24 equal parts, and includes all the parts but one: its value, therefore, very nearly equals that of the unit. $\frac{6}{6}$ expresses the division of an object into 6 equal parts, and includes all these parts, and consequently just equals the object or number divided. $\frac{7}{6}$ expresses more parts of the kind indicated by the denominator than can be obtained from any single object, and consequently its value is more than that of the unit. Hence, *with a given denominator*, the greater the numerator, *the greater the value of the fraction;* and the less the numerator, *the less the value of the fraction.*

§ **107.** We learn from the preceding paragraph, *that a fraction increases in value in proportion as its numerator is increased, while its denominator remains unchanged. Therefore, multiplying the numerator by any number, multiplies the value of the fraction by the same number.* Take the fraction $\frac{5}{6}$ as an illustration; if the numerator 5 be multiplied by 3, the 5 parts are increased to 15, while the value of each individual part is unchanged. We then have $\frac{15}{6}$, which is 3 times $\frac{5}{6}$.

§ **108.** If, on the other hand, *the numerator of a fraction be divided by any number, the fraction is divided by that number.* Taking half the parts given, divides the value expressed by 2; taking a third of those parts, takes $\frac{1}{3}$ their value.

§ **109.** *If we divide the denominator of a fraction by any number, we obviously multiply the value of a fraction by that number;* since the *number* of given parts remains unchanged, but *the value of each individual part is increased by making their number less.*

ILLUSTRATION: $\frac{10}{24} \times 2 = \frac{10}{12}$.

What is the value of $\frac{6}{6}$? Why? What is the value of $\frac{7}{6}$? Why? If the denominator be given, how is the value of a fraction?

107. In what proportion does a fraction increase in value? What then is the effect of multiplying the numerator?

108. What is the effect of dividing the numerator? Illustrate.

109. What is the effect of dividing the denominator? Why?

§ 110. If we multiply the denominator of a fraction by any number, *we divide the value of the fraction by that number*, since the number of given parts remains unchanged, but each individual part is diminished. Thus, $\frac{5}{8} \div 3 = \frac{5}{24}$, since $\frac{1}{24}$ of any object is only $\frac{1}{3}$ the value of its eighth part.

§ 111. If the numerator and denominator of a fraction are both multiplied or both divided by the same number, *the value of that fraction is not altered.*

ILLUSTRATION. $\frac{12}{3} = 4$; now if we multiply the terms of the fraction by 2 it becomes $\frac{24}{6}$, also $= 4$.

§ 112. From the foregoing remarks, it is obvious, that any change made in the terms of a fraction does not affect its value, so long as *the ratio of the numerator and denominator is not altered;* also that the value of a fraction increases in the same ratio as *its numerator increases*, while its denominator remains unchanged, or as *its denominator decreases*, while its numerator remains unchanged; and finally, that the value of a fraction *decreases* in the same ratio as its *numerator* decreases, while *its denominator remains the same*, or as the *denominator increases*, while the *numerator remains the same.*

REDUCTION OF FRACTIONS.

The reduction of fractions consists *in changing their forms*, while the *value* is not altered.

CASE I.

§ 113. TO REDUCE FRACTIONS TO THEIR LOWEST TERMS.

NOTE. Fractions are reduced to their lowest terms, when their numerators and denominators are the smallest that can be used and express their true value.

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110. What is the effect of multiplying the denominator? Why?

111. What is the effect of multiplying numerator and denominator both?

112. If the ratio of the numerator and denominator be not altered, what? In what ratio does the value of a fraction increase? In what ratio does the value of a fraction decrease?

113. What is reduction of fractions? When are fractions reduced to their lowest terms?

RULE.

Divide both numerator and denominator by any number greater than 1, *that will divide them both without remainder, so long as there is any number that will so divide them.*

1. Reduce $\frac{9}{12}$ to its lowest terms.

EXPLANATION. 9 the numerator and 12 the denominator are both divisible by 3 without remainders; hence

$$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}.$$

The terms now obtained, viz., numerator 3 and denominator 4, are not divisible by any number larger than 1, and are the smallest terms by which the value of $\frac{9}{12}$ can be expressed.

2. Reduce $\frac{24}{32}$ to its lowest terms.

$$\frac{24 \div 2 = 12}{32 \div 2 = 16}, \text{ and } \frac{12 \div 2 = 6}{16 \div 2 = 8} \text{ and } \frac{6 \div 2 = 3}{8 \div 2 = 4}, \textit{ Ans.}$$

It is better, however, first to divide by the largest number practicable, thus,

$$\frac{24 \div 8 = 3}{32 \div 8 = 4}.$$

Such a number may always be found, in the following manner:—1*st. Divide the greater of the given numbers by the less, and note the remainder. 2nd. Divide the smaller of the two numbers (the first divisor) by the remainder, and if there be still a remainder, divide the last divisor by that remainder, and so continue to do: that is, to divide the last divisor by the last remainder, till the division is effected without a remainder; the last divisor* will be the divisor required. Take the above sum as an example, $32 \div 24 = 1$ and 8 remainder; and $24 \div 8 = 3$ and no remainder. Hence, 8 is the divisor required, and

$$\frac{24 \div 8 = 3}{32 \div 8 = 4},$$

the same as above. This divisor is called the *greatest common measure* of those numbers.

What is the rule? How may the largest common divisor be found? *What is the* required divisor?

3. Reduce $\frac{16}{40}$ to its lowest terms, by dividing the given terms by their greatest common measure. *Ans.* $\frac{2}{5}$.

4. Reduce $\frac{60}{72}$ to its lowest term. *Ans.* $\frac{5}{6}$.

5. Reduce in like manner, $\frac{48}{60}$. *Ans.* $\frac{4}{5}$. Also $\frac{21}{45}$. *Ans.* $\frac{7}{15}$.

In most instances, however, especially if the terms of the fraction are small, it will be found more convenient, as well as more expeditious, to assume a divisor.

6. Reduce $\frac{9}{12}$ to its lowest terms. *Ans.* $\frac{3}{4}$.

7. Reduce $\frac{15}{25}$ to its lowest terms. *Ans.* $\frac{3}{5}$.

8. Reduce $\frac{18}{21}$ to its lowest terms. *Ans.* $\frac{6}{7}$.

9. Reduce $\frac{24}{36}$ to its lowest terms. *Ans.* $\frac{2}{3}$.

10. Reduce $\frac{48}{64}$ to its lowest terms, *Ans.* $\frac{3}{4}$.

CASE II.

§ 114. To reduce a Mixed Quantity to an Improper Fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and to the product, add the given numerator. The number thus produced, will be the numerator of the required fraction.

1. Reduce $16\frac{3}{4}$ to an improper fraction;—that is, find how many fourths there are in $16\frac{3}{4}$.

Explanation. Each unit=4 fourths; hence, 16 units= $16\times4=64$ fourths, and $64+3=67$ fourths, or $\frac{67}{4}$, *Ans.*

2. Reduce $17\frac{5}{6}$ to an improper fraction. *Ans.* $\frac{107}{6}$.

3. Reduce $21\frac{7}{8}$ to an improper fraction. *Ans.* $\frac{175}{8}$.

4. Reduce $7\frac{1}{11}$ to an improper fraction. *Ans.* $\frac{78}{11}$.

5. Reduce $6\frac{6}{7}$ to an improper fraction. *Ans.* $\frac{48}{7}$.

6. Reduce $9\frac{2}{3}$ to an improper fraction. *Ans.* $\frac{29}{3}$.

§ 115. Any whole number may be reduced to an improper fraction, by *multiplying it by the proposed denominator. The product* will form the numerator of the required fraction.

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114. How reduce a mixed quantity to an improper fraction?

115. How may a whole number be reduced to an improper fraction?

7. Reduce 8 to a fraction, whose denominator is 7.

EXPLANATION. $8\times7=56$, numerator; hence, $\frac{56}{7}$, *Ans.*

8. Reduce 9 to a fraction whose denominator is 11.
Ans. $\frac{99}{11}$.

9. Reduce $7\frac{3}{8}$ to an improper fraction. *Ans.* $\frac{59}{8}$.

10. Reduce $17\frac{4}{9}$ to an improper fraction. *Ans.* $\frac{157}{9}$.

CASE III.

§ **116.** TO REDUCE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER.

RULE.

Divide the numerator by the denominator. The quotient will be the whole number; if there be a remainder, write it over the denominator on the right of the whole number.

1. Reduce $\frac{156}{009}$ to a mixed number.

EXPLANATION. 9 ninths=1 whole one; therefore, $156\div9=17\frac{3}{9}$, *Ans.*

2. Reduce $\frac{73}{12}$ to a mixed number. *Ans.* $6\frac{1}{12}$.
3. Reduce $\frac{96}{8}$ to a whole number. *Ans.* 12.
4. Reduce $\frac{79}{8}$ to a mixed number. *Ans.* $9\frac{7}{8}$.
5. Reduce $\frac{171}{12}$ to a mixed number. *Ans.* $14\frac{3}{12}=14\frac{1}{4}$.
6. Reduce $\frac{198}{13}$ to a mixed number. *Ans.* $15\frac{3}{13}$.
7. Reduce $\frac{201}{070}$ to a mixed number. *Ans.* $28\frac{5}{7}$.

CASE IV.

§ **117.** TO REDUCE COMPOUND FRACTIONS TO SIMPLE FRACTIONS.

RULE.

Multiply the numerators of the several fractions together for a new numerator, and the denominators together for a new denominator. The fraction thus produced, when reduced to its lowest terms, (§ 113,) will be the one required.

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116. How reduce an improper fraction to a whole or mixed number?

117. How reduce compound fractions to simple ones?

1. Reduce $\frac{3}{4}$ of $\frac{6}{7}$ to a simple fraction. To obtain $\frac{1}{4}$ of $\frac{6}{7}$, we divide $\frac{6}{7}$ by 4; but $\frac{6}{7} \div 4 = \frac{6}{28}$, (§ **110**,) then having $\frac{1}{4}$ of $\frac{6}{7}$, if we multiply this by 3, we shall obtain $\frac{3}{4}$ of $\frac{6}{7}$; therefore, $\frac{6}{28} \times 3 = \frac{18}{28}$, the *Ans.* (§ **107**.) We have here, merely multiplied *numerators by numerators, and denominators by denominators.*

2. Reduce $\frac{1}{2}$ of $\frac{7}{8}$ to a simple fraction. *Ans.* $\frac{7}{16}$.

§ **118.** The solutions of sums of this description may be essentially abbreviated *by canceling such factors as are found common both to the numerators and denominators* of the several simple fractions.

3. Reduce $\frac{3}{4}$ of $\frac{5}{9}$ of $\frac{6}{7}$ to a simple fraction.

OPERATION: $\frac{\overset{2.}{\cancel{3}.}\ \overset{2}{\cancel{6}}}{\underset{2}{\cancel{4}.}\ \underset{3}{\cancel{9}.}\ 7}$; then $2 \times 2 = 4$ numerator, and 7 is the denominator; hence, $\frac{4}{7}$, *Ans.*

4. Reduce the fraction $\frac{5}{8}$ of $\frac{3}{15}$ of $\frac{6}{7}$.

Statement: $\frac{\overset{1}{\cancel{5}.}\ \overset{3}{\cancel{3}.}\ \overset{1}{\cancel{6}}}{\underset{4}{\cancel{8}.}\ \underset{1}{\cancel{15}.}\ 7} = \frac{3}{28}$, *Ans.*

5. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{10}$ to a simple fraction. *Ans.* $\frac{9}{16}$.
6. Reduce $\frac{2}{5}$ of $\frac{18}{19}$ of $\frac{8}{15}$ to a simple fraction. *Ans.* $\frac{16}{95}$.
7. Reduce $\frac{1}{3}$ of $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to a simple fraction. *Ans.* $\frac{1}{7}$.
8. Reduce $\frac{6}{7}$ of $\frac{11}{12}$ of $\frac{14}{15}$ to a simple fraction. *Ans.* $\frac{11}{15}$.

Whenever any term of a compound fraction is a *mixed number*, reduce it to an improper fraction. (§ **113**.)

9. Reduce $\frac{7}{8}$ of $\frac{6}{7}$ of $5\frac{1}{3}$ to a simple fraction.

EXPLANATION. $5\frac{1}{3} = \frac{16}{3}$. (§ **114**.) Hence, $\frac{7.\ 6.\ 16}{8.\ 7.\ 3}$, is the proper statement, $\frac{4}{1}$ or 4, *Ans.*

10. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $8\frac{1}{2}$ to a simple fraction.

Ans. $\frac{51}{16} = 3\frac{3}{16}$. (§ **116**.)

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118. How may such operations be abbreviated? What is to be done if a term of a compound fraction be a mixed number?

CASE V.

§ 119. To reduce Fractions to a Common Denominator.

The *value* of fractions is not altered by what is here done. All fractions are made to represent the unit *as divided into the same number of parts*, while the *number* of these parts differs, in each of the fractions. Thus, the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$, differ from each other both in *specific value*, and in the *number of parts*; but the fractions, $\frac{4}{8}$, $\frac{2}{8}$, and $\frac{1}{8}$, expressing the same value, differ in *specific value only*. These fractions now, therefore, *have the common denominator* 8.

RULE.

Multiply all the denominators together for a common denominator, and each numerator into all the denominators except its own, for a numerator to each new fraction.

1. Reduce $\frac{6}{7}$, $\frac{4}{5}$, and $\frac{8}{11}$ to a common denominator.

Operation. $7\times5\times11=385$, the common denominator.
$6\times5\times11=330$, numerator for fraction $\frac{6}{7}$.
$4\times7\times11=308$, numerator for fraction $\frac{4}{5}$.
$8\times5\times7=280$, numerator for fraction $\frac{8}{11}$.

Here the required fractions are, for $\frac{6}{7}$, $\frac{330}{385}$; for $\frac{4}{5}$, $\frac{308}{385}$; and for $\frac{8}{11}$, $\frac{280}{385}$.

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ to a common denominator.

Ans. $\frac{60}{120}$, $\frac{40}{120}$, $\frac{30}{120}$, $\frac{24}{120}$.

The value of these several fractions can, however, be exactly expressed in *smaller terms*, and *retain a common denominator*, for $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$ and $\frac{12}{60}$ are obviously of the *same value*, their *common denominator* being reduced from 120 to 60. It is always desirable to preserve the fractions in the *smallest possible terms* that will *express their true value*.

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119. Is the value of a fraction altered by the operation of this rule? In what does the change made consist? Illustrate? What is the rule? How may the value of the answers to sum 1st be expressed? *Illustrate?*

To do this, *first find the least common multiple of all the denominators, and make this the common denominator. Divide this common denominator by the denominator of each of the given fractions, and multiply the quotient by the numerators of the same.*

OBS. The least common multiple of two or more numbers *is the smallest number that can be divided by each of those numbers without remainder*, and is found as follows:

Write down the given numbers in a line from left to right, divide by the smallest number that will divide any two of them without remainder, and write the quotient and the undivided numbers in a line below the former numbers. Divide these numbers in turn in the same manner, and so on till no two numbers are left that can be divided by any number greater than 1. *The continued product of the divisors and of the undivided numbers of the lowest line, will be the multiple required.*

ILLUSTRATION. Required the least common multiplier of 4, 6, 8, and 12.

$$\begin{array}{r|l} 2 & 4.\ 6.\ 8.\ 12 \\ 2 & 2.\ 3.\ 4.\ \ 6 \\ 3 & 1.\ 3.\ 2.\ \ 3 \\ \hline & 1.\ 1.\ 2.\ \ 1 \end{array}$$

Then $2\times2\times3\times2=24$ is the least number that can be divided by each of the given numbers without remainder.

3. Reduce the fractions, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{9}$, and $\frac{3}{10}$, to their least common denominator.

$$\begin{array}{r|l} 2 & 2.\ 3.\ 9.\ 10 \\ 3 & 1.\ 3.\ 9.\ \ 5 \\ \hline & 1.\ 1.\ 3\ \ \ 5 \end{array}$$

; and $2\times3\times3\times5=90$, the least common multiple of the denominators, therefore,

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How may the least common denominator always be found? What is the least common multiple of two or more numbers? How is it found?

$\overline{90 \div 2} \times 1 = 45$, 1st numerator, Hence, $\frac{45}{90}$ is the 1st fraction.
$\overline{90 \div 3} \times 2 = 60$, 2d numerator, and $\frac{60}{90}$, the 2d fraction.
$\overline{90 \div 9} \times 8 = 80$, 3d numerator, and $\frac{80}{90}$, the 3d fraction.
$\overline{90 \div 10} \times 3 = 27$, 4th numerator, and $\frac{27}{90}$, the 4th fraction.
Hence, $\frac{45}{90}$, $\frac{60}{90}$, $\frac{80}{90}$, and $\frac{27}{90}$, are the fractions required.

4. Reduce $\frac{1}{3}$, $\frac{5}{6}$, $\frac{4}{9}$, and $\frac{7}{12}$, to their least common denominator. *Ans.* $\frac{12}{36}$, $\frac{30}{36}$, $\frac{16}{36}$, and $\frac{21}{36}$.

5. Reduce $\frac{3}{5}$, $\frac{1}{3}$, $\frac{7}{9}$, and $\frac{8}{11}$, to their least common denominator. *Ans.* $\frac{297}{495}$, $\frac{165}{495}$, $\frac{385}{495}$, and $\frac{360}{495}$.

6. Reduce $\frac{1}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{7}{8}$, to their least common denominator. *Ans.* $\frac{96}{120}$, $\frac{100}{120}$, $\frac{90}{120}$, and $\frac{105}{120}$.

7. Reduce $\frac{3}{8}$, $\frac{3}{4}$, $\frac{6}{7}$, and $1\frac{3}{14}$, to their least common denominator. *Ans.* $\frac{21}{56}$, $\frac{42}{56}$, $\frac{48}{56}$, and $\frac{52}{56}$.

8. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$, to their least common denominator. *Ans.* $\frac{27}{36}$, $\frac{30}{36}$, and $\frac{28}{36}$.

9. Reduce $\frac{5}{12}$, $\frac{7}{8}$, and $\frac{8}{9}$, to their least common denominator. *Ans.* $\frac{30}{72}$, $\frac{63}{72}$, and $\frac{64}{72}$.

10. Reduce $\frac{5}{16}$, $\frac{1}{8}$, $\frac{7}{12}$, and $\frac{5}{9}$, to their least common denominator. *Ans.* $\frac{45}{144}$, $\frac{18}{144}$, $\frac{84}{144}$, and $\frac{80}{144}$.

ADDITION OF FRACTIONS.

§ **120.** Like things only can be added, so as to form one number. (§ **32**, 4.) Hence, before adding fractions *they must have a common denominator.*

RULE.

If the denominators of the given fractions are unlike, first reduce them to a common denominator, and, adding the numerators, place their sum over the common denominator.

1. Add the fractions $\frac{3}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, and $\frac{7}{8}$ together. These fractions have the common denominator 8; hence $3+5+6+7=21$, the sum of the numerators, and $\frac{21}{8}$ is the required answer.

120. What things can be added? What must fractions have *before they* can be added together? What is the rule?

2. Add the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. These fractions have different denominators. Therefore, (§ **119**, Obs.)

$$\begin{array}{r} 2)2.3.4.5 \\ \hline 1.3.2.5 \end{array}$$; $5\times2\times3\times2=60$ least common denominator.

The fractions are, $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, and $\frac{12}{60}$; and $30+20+15+12=77$, the sum of the numerators; hence $\frac{77}{60}$, *Ans.*

3. Add together the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.

Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, and $\frac{50}{60}$.

4. What is the sum of $\frac{4}{5}$, $\frac{6}{7}$, $\frac{3}{8}$, and $\frac{9}{14}$?

Ans. $\frac{149}{280}=2\frac{189}{280}$. (§ **116**.)

5. What is the sum of $\frac{5}{6}$, $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{11}{12}$?

Ans. $\frac{363}{120}=3\frac{3}{120}=3\frac{1}{40}$. (§ **113**.)

6. What is the sum of $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{1}{4}$? *Ans.* $\frac{287}{120}=2\frac{47}{120}$.

7. What is the amount of $\frac{7}{8}$, $\frac{5}{6}$, $\frac{4}{9}$, and $\frac{8}{9}$?

Ans. $\frac{219}{72}=3\frac{3}{72}=3\frac{1}{24}$.

8. What is the amount of $\frac{20}{21}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{3}{14}$?

Ans. $\frac{231}{84}=2\frac{63}{84}=2\frac{3}{4}$.

9. What is the amount of $4\frac{1}{3}$, $6\frac{1}{6}$, $1\frac{4}{9}$, and $\frac{5}{12}$?

Ans. $\frac{345}{36}=12\frac{13}{36}$.

10. What is the amount of $3\frac{1}{4}$, $5\frac{3}{8}$, $11\frac{1}{12}$, and $\frac{1}{8}$?

Ans. $\frac{237}{24}=9\frac{21}{24}=9\frac{7}{8}$.

SUBTRACTION OF FRACTIONS.

§ **121.** Arithmetical quantities can be subtracted *only when they agree in kind.* Hence, in subtracting fractions:

RULE.

Reduce the given fractions to a common denominator, if necessary, and subtract the numerators.

1. From $\frac{11}{12}$ take $\frac{7}{12}$. These fractions have a common denominator, therefore $11-7=4$, the difference of numerators; and $\frac{4}{12}=\frac{1}{3}$, *Ans.*

2. From $\frac{19}{20}$ take $\frac{7}{20}$. *Ans.* $\frac{12}{20}=\frac{3}{5}$.

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121. In what respect must quantities agree to be subtracted? *What is the rule?*

3. From $\frac{15}{16}$ take $\frac{11}{12}$. First find a common denominator.

2) 16. 12

2) 8. 6

4. 3; and $2\times2\times4\times3=48$, least common denominator.

The fractions, then, are $\frac{45}{48}$ and $\frac{44}{48}$; and $45-44=1$. Hence, $\frac{1}{48}$, *Ans.*

4. What is the difference between $\frac{6}{7}$ and $\frac{7}{8}$? *Ans.* $\frac{1}{56}$.
5. What is the difference between $\frac{21}{36}$ and $\frac{7}{12}$? *Ans.* 0.
6. What is the difference between $3\frac{5}{6}$ and $2\frac{1}{2}$? *Ans.* $\frac{5}{6}$.
7. What is the difference between $1\frac{3}{5}$ and $\frac{7}{16}$? *Ans.* $1\frac{5}{16}$.
8. What is the difference between $4\frac{2}{4}$ and $3\frac{7}{8}$? *Ans.* $\frac{1}{8}$.
9. What is the difference between $1\frac{6}{7}$ and $\frac{5}{8}$? *Ans.* $\frac{43}{136}$.
10. What is the difference between $\frac{21}{25}$ and $\frac{8}{11}$? *Ans.* $\frac{31}{275}$.

MULTIPLICATION OF FRACTIONS.

§ **122.** The multiplication by fractions naturally resolves itself into three distinct operations, viz.: *the multiplication of a fraction by a whole number; of a whole number by a fraction; and of a fraction by a fraction.*

§ **123.** To Multiply a Fraction by a Whole Number.

RULE.

Multiply the numerator of the fraction by the whole number, (§ **107**;) or, if practicable, without remainder, *divide the denominator by that number.* (§ **109**.)

1. Multiply $\frac{3}{4}$ by 5. $\frac{3}{4}\times5=\frac{15}{4}$, 5 times 1 fourth of a number=5 fourths of the same; 5 times 3 fourths must then=3 times 5 fourths=$\frac{15}{4}$. *Ans.* $\frac{15}{4}=3\frac{3}{4}$. (§ **116**.)

In the operations here given, it is obvious that any factor *common to the multiplier and the denominator of the fraction, may be canceled.*

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122. Into what does the multiplication of fractions naturally resolve itself?

123. Rule for multiplying a fraction by a whole number? What *may* be done with any factor common to the multiplier and the denominator,

2. Multiply $\frac{5}{9}$ by 12. Here 9, the denominator, and 12, the whole number, contain each the common factor, 3.

Therefore, $\frac{5.\ \overset{4}{\cancel{12}}}{\underset{3}{\cancel{9}}}=5\times4=20$, that is, $\frac{20}{3}=6\frac{2}{3}$.

A fraction is multiplied into a number equal to its denominator *by canceling that denominator.*

3. Multiply $\frac{7}{8}$ by 8.

OPERATION. $\frac{7.\ \cancel{8}}{\cancel{8}}=\frac{7}{1}=7.$

4. Multiply $\frac{16}{21}$ by 9. *Ans.* $4\frac{8}{7}$, or $6\frac{2}{7}$.
5. Multiply $\frac{23}{27}$ by 18. *Ans.* $15\frac{1}{3}$.
6. Multiply $\frac{25}{28}$ by 21. *Ans.* $18\frac{3}{4}$.
7. Multiply $\frac{19}{30}$ by 15. *Ans.* $9\frac{1}{2}$.
8. Multiply $\frac{16}{17}$ by 5. *Ans.* $4\frac{12}{17}$.
9. Multiply $\frac{31}{35}$ by 21. *Ans.* $18\frac{3}{5}$.
10. How much will 17 pounds of tea cost, at $\frac{3}{4}$ of a dollar a pound? *Ans.* $12\frac{3}{4}$.
11. How much will 27 bushels of wheat cost, at $\frac{8}{9}$ of a dollar a bushel? *Ans.* $24.
12. How much will 15 bushels of rye cost, at $\frac{3}{10}$ of a dollar a bushel? *Ans.* $4\frac{1}{2}$.

§ **124.** TO MULTIPLY A WHOLE NUMBER BY A FRACTION.

A fraction is *a part* of a unit. Multiplying by a fraction must, therefore, produce a number *less than the one multiplied.* But the denominator of the given fractional multiplier *shows what part of a given number is repeated,* and the numerator, *the number of repetitions required.* Hence,

RULE.

Divide the given number by the denominator of the fractional multiplier, and multiply the quotient by the numerator.

124. What does the denominator of a fractional multiplier show? *What the numerator?* What is the rule?

1. If a pound of beef cost 9 cents, what is $\frac{2}{3}$ of a pound worth?

1 third of a thing taken twice is 2 thirds of that thing. Hence, $9 \div 3 = 3$ cents = the price of 1 third of a pound; $3 \times 2 = 6$ cents, the price of 2 thirds, *Ans.*

The above sum canceled; $\frac{\overset{3}{\cancel{9}} \cdot 2}{\underset{1}{\cancel{3}}}$, and $3 \times 2 = 6$ cents, as before; that is, we *write the whole number and the numerator of the fraction above a horizontal line, and the denominator below the same*, and cancel.

2. If a hundred pounds of beef cost \$12, what will $\frac{3}{4}$ of a hundred pounds cost? *Ans.* \$9.

3. If a man earn \$30 in a month, how much will he earn in $\frac{5}{6}$ of a month? *Ans.* \$25.

4. If a ton of hay cost \$24, what is the value of $\frac{5}{8}$ of a ton? *Ans.* \$15.

5. If a bushel of wheat cost $\frac{5}{8}$ of a dollar, what will 44 bushels cost? *Ans.* \$27.50.

6. If a barrel of flour be worth \$9, what is $\frac{7}{8}$ of a barrel worth? *Ans.* $\$7\frac{7}{8}$.

7. If a hogshead of wine be worth \$48, what will $\frac{4}{5}$ of a hogshead cost? *Ans.* $\$38\frac{2}{5}$.

8. If an acre of land produce 148 bushels of oats, how many bushels will $\frac{5}{6}$ of an acre produce?

Ans. $123\frac{1}{3}$ bushels.

9. If one bushel of seed produce 88 bushels of corn, how many bushels will $1\frac{1}{2}$ of a bushel produce?

Ans. $80\frac{2}{3}$ bushels.

§ 125. To Multiply Fractions by Fractions.

If the multiplicand be a fraction, that is, *a part* of a unit, and if the multiplier be also a fraction, implying that *a part only* of this part is required; the product must be less than the multiplicand. The same is always the case,

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How cancel?

125. In the multiplication of fractions, how does the product compare *with the* multiplicand?

whenever the multiplier is a fraction *of less value than a unit.*

RULE.

Multiply the numerators of the fractions together for the numerator, and the denominators for the denominator of the answer.

1. Multiply $\frac{4}{5}$ by $\frac{3}{4}$. $\frac{12}{20}=\frac{3}{5}$. (§ 113.)

The labor of reducing the product to lower terms, will be avoided by *canceling all the factors common to both numerators and denominators.*

If the above sum be solved by this rule, the numerator 4 will cancel the denominator 4, and leave the answer $\frac{3}{5}$.

2. Multiply $\frac{4}{7}$ by $\frac{3}{8}$. *Ans.* $\frac{3}{14}$.
3. Multiply $\frac{5}{9}$ by $\frac{7}{20}$. *Ans.* $\frac{7}{36}$.
4. Multiply $\frac{7}{8}$ by $\frac{8}{9}$. *Ans.* $\frac{7}{9}$.
5. Multiply $\frac{15}{16}$ by $\frac{4}{7}$. *Ans.* $\frac{15}{28}$.
6. A man owning $\frac{3}{4}$ of a vessel, sold $\frac{5}{6}$ of his share; what part of the whole ship did he sell? *Ans.* $\frac{5}{8}$.
7. A farmer bought $\frac{3}{4}$ of a lot of land, and sold $\frac{7}{8}$ of his purchase; what part of the whole lot did he sell? *Ans.* $\frac{21}{32}$.
8. Multiply $\frac{5}{9}$ by $\frac{18}{10}$. *Ans.* $\frac{10}{10}$.
9. Multiply $\frac{21}{25}$ by $\frac{6}{14}$. *Ans.* $\frac{18}{25}$.
10. Multiply $\frac{6}{7}$ by $\frac{5}{12}$. *Ans.* $\frac{5}{14}$.
11. Multiply $\frac{13}{14}$ by $\frac{21}{26}$. *Ans.* $\frac{3}{4}$.

DIVISION OF FRACTIONS.

§ 126. The division of fractions is also resolved into three distinct processes, viz.: the division of a fraction by a whole number; the division of a whole number by a fraction; and the division of a fraction by a fraction.

What is the rule? How may we avoid the labor of reduction?
126. Into how many processes is the division of fractions divided? What are they?

§ 127. To Divide a Fraction by a Whole Number.

RULE.

Divide the numerator by the whole number if practicable; if not, multiply the denominator by the same.

1. What is the quotient of $\frac{8}{9}$ divided by 4? *Ans.* $\frac{2}{9}$.
2. What is the quotient of $\frac{3}{5}$ divided by 8? *Ans.* $\frac{3}{40}$.
3. What is the quotient of $\frac{4}{7}$ divided by 7? *Ans.* $\frac{4}{49}$.
4. If $\frac{7}{8}$ of an acre of land produce 28 bushels of corn, what fraction of an acre will produce 1 bushel? *Ans.* $\frac{1}{32}$.
5. If $\frac{5}{6}$ of a load of hay be valued at \$15, what fraction of a load can be bought for \$1? *Ans.* $\frac{1}{18}$ of a load.
6. If $\frac{1}{16}$ of a ton of iron be valued at \$12, what fraction of a ton can be bought for \$1? *Ans.* $\frac{5}{64}$ of a ton.

§ 128. To Divide a Whole Number by a Fraction.

In dividing a whole number by a fraction, the fraction being *less than a unit*, will be contained in the dividend *more times than it contains units*. Hence, the *quotient will be larger* than the dividend, unless the divisor be *an improper fraction*.

RULE.

Multiply the whole number by the denominator of the fraction, and divide the product by the numerator of the same.

1. Divide 21 by $\frac{7}{8}$. Here it is obvious that 21 is to be reduced to eighths, and these eighths be divided by 7. This result will be obtained by multiplying by the fractional divisor inverted. Hence, $21 \times \frac{8}{7} = 24$, *Ans.*

We may, therefore, invert the fractional divisor, and cancel equal factors.

2. Divide 18 by $\frac{3}{7}$.

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127. What is the rule for dividing a fractional number by a whole number?

128. How will the quotient, in all cases, compare with the dividend? *Why?* What is the rule? How do we divide by a fraction?

Canceled: $\frac{\overset{2}{\cancel{12}}. 7}{\underset{1}{\cancel{3}}}$, and $6 \times 7 = 42$, *Ans.*

3. Divide 75 by $\frac{5}{9}$. *Ans.* 135.
4. Divide 84 by $\frac{6}{7}$. *Ans.* 98.
5. Divide 18 by $\frac{5}{14}$. *Ans.* $50\frac{2}{5}$.
6. Divide 33 by $\frac{11}{2}$. *Ans.* 36.
7. Divide 71 by $\frac{2}{4}$. *Ans.* $94\frac{2}{4}$.
8. Divide 17 by $\frac{9}{7}$. *Ans.* $19\frac{3}{9}$.

§ **129.** To Divide a Fraction by a Fraction.

RULE.

Invert the divisor and then proceed as in multiplying fractions by fractions.

1. Divide the fraction $\frac{7}{8}$ by $\frac{2}{3}$.

Performed. $\frac{7}{8} \times \frac{3}{2} = \frac{21}{16} = 1\frac{5}{16}$, *Ans.*

Explanation. A unit is contained in $\frac{7}{8}$, 7 eighths of a time, consequently $\frac{1}{8}$ of a unit is contained in the same $\frac{7}{8}$, three times as often, $= \frac{7}{8} \times 3 = \frac{21}{8}$ times. But $\frac{2}{3}$ is contained $\frac{1}{2}$ as many times as $\frac{1}{3}$, and $\frac{1}{2}$ of $\frac{21}{8} = \frac{21}{16} = 1\frac{5}{16}$, *Ans.* (§ **116**.)

2. Divide $\frac{6}{7}$ by $\frac{3}{5}$. *Ans.* $\frac{10}{7}$ or $2\frac{3}{7}$.
3. Divide $\frac{12}{3}$ by $\frac{9}{7}$. *Ans.* $\frac{14}{3}$ or $1\frac{1}{3}$.
4. Divide $\frac{16}{17}$ by $\frac{8}{11}$. *Ans.* $\frac{22}{17}$ or $1\frac{5}{17}$.
5. Divide $\frac{10}{21}$ by $\frac{2}{7}$. *Ans.* $\frac{10}{6}$ or $1\frac{1}{3}$.
6. Divide $\frac{1}{2}$ by $\frac{23}{24}$. *Ans.* $\frac{12}{23}$.

Observation. If either the divisor or dividend be a compound fraction, *all the terms of that compound fraction must be written in the same manner;* that is, *all the numerators of the dividend and all the denominators of the divisor, must be placed above the line.*

7. Divide $\frac{7}{8}$ of $\frac{5}{9}$ by $\frac{15}{24}$.

Terms arranged: $\frac{7. 5. 24}{8. 9. 15}$. Factors common to the numerators and denominators, may be rejected.

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129. How divide a fraction by a fraction? If either the dividend or divisor be a compound fraction, what must be done?

Hence, $\frac{7.\ \cancel{5}.\ \cancel{24}^{\cancel{3}}}{\cancel{8}.\ 9.\ \cancel{15}^{\cancel{5}}} = \frac{7}{5}$, *Ans.*

8. Divide $\frac{5}{6}$ of $\frac{9}{10}$ by $\frac{3}{4}$ of $\frac{6}{7}$.

Terms arranged: $\frac{5.\ 9.\ 4.\ 7}{6.\ 10.\ 3.\ 6}$. Canceled: $\frac{\cancel{5}.\ \cancel{9}.\ \cancel{4}.\ 7}{\cancel{6}.\ \cancel{10}.\ \cancel{3}.\ 6}$;

therefore $\frac{7}{6} = 1\frac{1}{6}$ is the *Ans.*

9. Divide $\frac{3}{8}$ of $\frac{11}{12}$ by $\frac{1}{6}$ of $\frac{13}{24}$. *Ans.* $\frac{99}{26}$ or $3\frac{21}{24}$.
10. Divide $\frac{1}{6}$ of $\frac{5}{9}$ by $\frac{5}{6}$ of $\frac{3}{7}$. *Ans.* $\frac{7}{27}$.
11. Divide $\frac{3}{4}$ of $\frac{11}{12}$ by $\frac{46}{48}$. *Ans.* $\frac{33}{46}$.
12. Divide $\frac{17}{20}$ by $\frac{4}{5}$ of $\frac{3}{7}$. *Ans.* $\frac{119}{48} = 2\frac{23}{48}$.
13. Divide $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{8}$. *Ans.* $\frac{4}{1}$ or 2.
14. Divide $\frac{14}{15}$ of $\frac{15}{16}$ by $\frac{7}{8}$ of $\frac{9}{10}$. *Ans.* $1\frac{1}{9}$.

GENERAL APPLICATION.

1. What are the lowest terms in which $\frac{27}{36}$ can be expressed? *Ans.* $\frac{3}{4}$. (§ **113.**)
2. Reduce $17\frac{1}{5}$ to an improper fraction. *Ans.* $\frac{86}{5}$. (§ **114.**)
3. Reduce $\frac{78}{9}$ to a mixed number.
Ans. $8\frac{6}{9} = 8\frac{2}{3}$. (§ **116**, also **113.**)
4. Reduce $\frac{1}{2}$ of $\frac{8}{9}$ of $\frac{4}{5}$ of $\frac{18}{19}$ to a simple fraction?
Ans. $\frac{32}{95}$. (§ **117.**)
5. Reduce $\frac{3}{4}$ of $\frac{5}{4}$ of $\frac{1}{3}$ of $\frac{9}{10}$ to a simple fraction.
Ans. $\frac{9}{32}$.
6. Reduce $\frac{1}{5}$, $\frac{6}{7}$, $\frac{9}{10}$ and $\frac{9}{14}$, to a common denominator.
Ans. $\frac{14}{70}$, $\frac{60}{70}$, $\frac{63}{70}$, and $\frac{45}{70}$. (§ **119.**)
7. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{7}{12}$ to a common denominator.
Ans. $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$, $\frac{14}{24}$.
8. Add together the fractions, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{9}$, and $\frac{7}{8}$.
Ans. $\frac{221}{72} = 3\frac{5}{72}$. (§ **116.**)
9. From $1\frac{1}{2}$ subtract $\frac{8}{9}$. *Ans.* $\frac{3}{108} = \frac{1}{36}$. (§ **119**, **113.**)
10. Multiply $\frac{7}{8}$ by $\frac{15}{16}$. *Ans.* $\frac{105}{128}$. (§ **125.**)
11. Divide $\frac{3}{4}$ of $\frac{8}{9}$ by $\frac{21}{24}$. *Ans.* $\frac{16}{21}$. (§ **129.**)
12. Divide $\frac{4}{17}$ of $\frac{6}{7}$ by $\frac{2}{3}$ of $\frac{4}{5}$. *Ans.* $\frac{90}{17} = 1\frac{13}{17}$.

PER CENTAGE.

§ **130.** One individual often buys or sells merchandise and other property for another, receiving as a compensation for the same, *a certain number of cents for every dollar's worth of property bought or sold.* The money thus received is called *per centage.*

Hence, the *amount* of per centage received or paid *is determined by multiplying that value by the per centage.*

Obs. The expression *per cent.*, used in the sums of this section, means *by the hundred.* Thus, if 3 cents be paid on 100 cents, the rate is 3 per cent., &c.

1. What is the amount of per centage for selling $1200 worth of property at 3 per cent premium?

Operation. $1200 × .03. = $36.00.

It will be observed, that the amount here paid, is 3 cents for $1, and consequently the amount of per centage is as many times 3 cents as there are dollars, viz.: $1200 × .03 = 3600 cents = $36.

2. What is 5 per cent of $7?

5 per cent. is 5 cents for each dollar; hence,
5 × 7 = 35 cents, *Ans.*

3. What is 3 per cent. of $5? of $8? of $6? of $12?
4. What is 4 per cent. of $3? of $8? of $12? of $6?
5. What is 2 per cent. of $5? of $12? of $8? of $9?
6. What is 5 per cent. of $7? of $9? of $10? of $8?
7. What is 6 per cent. of $12? of $11? of $10?
8. What is 2 per cent. of $12? of $13? of $15?
9. What is 3 per cent. of $12? of $14? of $13?

§ **131.** One man is often employed to buy or sell goods for other men. When so employed, he is called *an agent or factor*, and receives for his services *a certain per cent. of the value of the goods bought or sold.* The amount received

130. How does one person often receive pay for buying or selling for another? What is the money thus received called? How is the amount of per centage determined? What does per cent. mean?

131. How is one man often employed? What is he then called? How paid?

by him is determined *by multiplying the value of his purchase or sale, by the specified per cent.* Goods so procured or disposed of, are said *to be bought or sold on commission.* The money paid the agent is called *his commission.*

RULE.

Multiply the money given, by the per centage. The multiplier will be a decimal of two places when it is expressed by cents only, and of three places when expressed by mills.

1. What is the per centage for selling $785.75 worth of goods, at 3 per cent. commission?

PERFORMED. $785.75 \times .03 = \$23.5725$; that is, $23. 57 cents. Mills, unless more than 5, are seldom regarded.

2. What is the commission for buying a house valued at $4720, at 2 per cent.? *Ans.* $94.40.

3. What is the commission for selling a farm valued at $9683, at 1 per cent.? *Ans.* $96.83.

4. What is 4 per cent. of $1728? *Ans.* $69.12.

5. What is 4 per cent. of $3786.79? *Ans.* $151.47+.

6. What is 5 per cent. of $879.68? *Ans.* $43.98+.

§ **132.** The insurance of property from loss by fire, storm, or otherwise, *is always computed at some certain rate per cent. upon the value of property destroyed. Hence, multiply the value of the property by the rate per cent. of insurance.*

1. What is the expense of insuring a house valued at $1600, at 2 per cent.? *Ans.* $32.

2. What will it cost to insure a house worth $2250, against loss by fire, at 3 per cent. premium? *Ans.* $67.50.

3. What is the cost of insuring a vessel and cargo valued at $45867, at 2 per cent.? *Ans.* $917.34.

4. What will it cost to insure property valued at $7963, at 5 per cent.? *Ans.* $398.15.

How is the money he receives determined? How are goods so procured said to be bought or sold? What is the money paid the agent called? What is the rule for commission?

132. How is insurance always computed? How determine the *amount to be paid* for insurance?

5. What is the premium for the insurance of property valued at $8864, at 6 per cent. premium? *Ans.* $531.84.

6. What is the premium for the insurance of a ship and cargo valued at $20500, at $\frac{3}{4}$ of 1 per cent. *Ans.* $153.75. (§ 124.)

INTEREST.

§ 133. Interest is an allowance made for the use of money. It is always computed at some given rate per cent. per annum; that is, *at the rate of some specified number of dollars for the use of a hundred dollars for one year*. If $6 be paid for the use of $100, the rate is said to be 6 per cent.; if $7 be paid for the same money, the rate is 7 per cent., &c.

If the time be more than *one year*, the rate per cent. is proportionably increased; that is, if the rate for one year be $6, for two years it is $6×2=$12; for three years, $6×3=$18; &c.

Observe the difference between interest as here explained, and commission; the latter being *a per centage on money, without regard to time;* the former, *a certain per cent. for the use of money for a limited time*, viz.; *for a year*.

Hence, the *amount of interest* payable for the use of any sum of money, depends both on *the amount of money*, and *on the time*.

1. If the interest of $1 for 1 year is 6 cents, what is the interest of $2? of $3? of $4?

2. If the interest of $1 for 1 year is 6 cents, what is it for 2 years? for 3 years? for 4 years?

3. If the interest of $1 for 1 year is 6 cents, what is it for $2 for 1 year? for 2 years? for 3 years?

4. If the interest of $1 for 1 year is 6 cents, what is the interest of $3 for 2 years? 3 years?

133. What is interest? How computed? What is to be understood by a given per cent.? If the time be more than a year, how is the rate affected? What is the difference between interest and commission? On what does the amount of interest payable at any one time depend?

5. If the interest of $1 for a year be 4 cents, what is it for $2, for 2 years? $2, for 3 years?

6. What is the interest of $6, for 2 years, at 3 per cent.?

7. What is the interest of $8, for 5 years, at 4 per cent.?

8. What is the interest of $5, for 2 years, at 7 per cent.?

9. What is the interest of $6, for 3 years, at 5 per cent.?

§ **134.** The following particulars require to be carefully noted.

1. The money on which interest is paid, is called *the principal.*

2. The money paid for the use of money, is called the *interest.* Interest is usually regulated by law, and when so regulated, it is called *legal interest.* The legal per centage differs in the different states. In the New England states, as well as in many others, it is 6 per cent.; in New York, it is 7 per cent.; in Louisiana, 8 per cent.; and in Texas, 10 per cent.

3. The *amount* of any sum drawing interest, *is found by adding together the principal and interest.*

4. The rate per cent. is always a decimal of *two places*, when expressed by cents, and of *three places* when expressed by mills. Hence, in writing down any per cent. less than 10, *a cipher must be placed before the figure*, since cents are hundredths of dollars. (§ **85.**)

§ **135.** To find the interest on any number of dollars for 1 year, *multiply the dollars by the rate per cent.; and, for a number of years, multiply the interest of* 1 *year by the number of years.*

1. What is the interest of $32, at 6 per cent., for 2 years?
Ans. $3.84. (§ **91.**)

2. What is the interest of $156, for 3 years, at 5 per cent.?
Ans. $23.40.

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134. What is the principal? What is the interest? How regulated? What then called? What is legal interest in New England? In Louisiana what? In Texas what? How is the amount of any sum drawing interest found? What is the per cent. when expressed by cents? When expressed by mills? In writing a per cent. less than 10, what must be done? Why?

135. How find interest for 1 year? For a number of years?

3. What is the interest of $355, for 4 years, at 3 per cent.? *Ans.* $42.60.

4. What is the interest of $786, for 3 years, at 6 per cent.? *Ans.* $141.48.

To find the *amount of principal and interest, that is, the sum due for a sum of money on interest, add the interest to the principal.*

5. What is the amount of $317.16, for 3 years, at 7 per cent.? *Ans.* $383.763.

6. What is the amount of $118.71, for 4 years, at 5 per cent.? *Ans.* $142.45.

7. What is the amount of $2163.20, for 3 years, at 6 per cent.? *Ans.* $2552.576.

8. What is the amount of $982.78, for 5 years, at 9 per cent.? *Ans.* $1425.031.

§ **136.** The per cent. at which interest is more usually cast, is 6 per cent.; that is, at the rate of 6 cents for the use of 100 cents, or $1, for one year. But a year is 12 months, and 6 cents for 12 months is just ½ a cent a month. Hence, the per cent. for any period of time, computing at the rate of 6 per cent. per annum, is always the same *as the half of the number of months in that time.*

1. What is the rate per cent. for 1 year and 6 months, the rate per annum being 6?

EXPLANATION. 1 year and 6 months=18 months, and 18 months÷2=9, which is the per cent. required, viz.: .09 per cent. (§ **134**, 4.)

2. What is the rate per cent. for 1 year and 10 months, the rate per annum being 6 per cent? for 1 year and 8 months? for 2 years and 4 months? for 2 years and 6 months? for 2 years and 10 months? for 3 years and 2 months? for 3 years and 6 months? for 3 years and 10 months? for 3 years and 4 months? for 3 years and 9 months? for 4 years and 7 months? for 4 years and 2

How find the amount due?

136. What is the more common per centage? What is that a month?

months? for 4 years and 8 months? for 4 years and 4 months? for 4 years and 10 months?

§ **137.** Since at 6 per cent. per annum, the interest of $1 for one month is ½ cent=5 mills, (§ **136**;) and since, in interest, 30 days are always allowed for a month, it requires just 6 days for one dollar to gain 1 mill. Hence the per cent. for any number of years, months, and days, at 6 per cent., is always found *by taking one half of the months in the given years and months, for cents, and one sixth part of the days, for mills.*

If there be an *odd month*, 5 mills must be added to the mills obtained from the given days, if any.

1. What is the per centage on $1, for 1 year, 8 months, and 12 days?

EXPLANATION. 1 year and 8 months=20 months, which equals the *half* cents of interest for each dollar of principal for 1 year and 8 months; and, hence, 20÷2=10, the interest for the whole time in cents; and 12 days÷6=2, the mills for the given days; and 10 cents+2 mills=.102, is the per cent. required for the whole time.

2. What is 6 per cent. of $1, for 2 years, 4 months, and 18 days? *Ans.* .143.

2 years, 4 months=28 months=14 cents; and 18 days÷6=3 mills; united=.143.

3. What is 6 per cent. of $1, for 2 years, 10 months, and 24 days? *Ans.* .174.

4. What is 6 per cent. of $1, for 3 years, 6 months, and 12 days? *Ans* .212.

5. What is 6 per cent. of $1, for 3 years, 9 months, and 6 days? *Ans.* .226.

Here we have an odd month for which we allow 5 mills, and the 1 mill obtained from the 6 days, gives 6 as the required number of mills.

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137. How many days allowed to the month in interest? In what time will a dollar gain a mill of interest? How is the rate for any *number* of years, months, and days found? What must be allowed *for an odd month*?

6. What is 6 per cent. of $1, for 3 years, 5 months, and 18 days? *Ans.* .208.

7. What is 6 per cent. of $1, for 2 years, 3 months, and 21 days? *Ans.* .138½.

The days given in this sum, when divided by 6, give 3½ as the mills for that number of days, which added to 13 cents, 5 mills, (.135) obtained from 2 years and 3 months, gives 13 cents, 8½ mills, (.138½,) as the required per centage for the given time. Hence, there will always be a fraction of a mill in the per centage of $1, for any specified time, *whenever the days in that time are not divisible by* 6 *without remainder.*

8. What is 6 per cent. of $1, for 3 years, 6 months, and 27 days? *Ans.* .214½.

9. What is 6 per cent. of $1, for 1 year, 9 months, and 9 days? *Ans.* .106½.

10. What is 6 per cent. of $1, for 3 years, 1 month, and 22 days? *Ans.* .188⅔.

11. What is 6 per cent. of $1, for 4 years, 7 months, and 13 days? *Ans.* .277⅙.

12. What is 6 per cent. of $1, for 2 years, 11 months, and 11 days? *Ans.* .176⅚.

13. What is 6 per cent. of $1, for each of the following periods of time: for 2 years, 3 months, and 18 days? for 1 year, 6 months, and 27 days? for 3 years, 2 months, and 24 days? for 1 year, 7 months, and 21 days?

If the given days are less than 6, the interest for those days will be *as many sixths of a mill* as there are days; thus, 1 day is ⅙ of a mill; 3 days, ⅗=½ of a mill; 5 days, ⅚ of a mill, &c.

14. What is 6 per cent of $1, for 3 years, 3 months, and 3 days? *Ans.* .195½.

15. What is 6 per cent. of $1, for 1 year, 10 months, and 5 days? 2 years, 7 months, and 1 day? 3 years, 1 month, and 4 days?

When will there be a fraction in the per centage obtained? If the *days are less than* 6, what will be the interest?

§ **138.** The following general rule will be found applicable in casting interest on any sum of money, and for any period of time. It must, however, be remembered that this rule is restricted to interest at 6 per cent. per annum.

RULE.

Find the per centage of \$1, *for the given time, as in the preceding section.* (§ **137**.) *Multiply the given money by this per centage written as a decimal, and point off decimals in the product, the same as in multiplication of decimals.* (§ **88**.)

1. What is the interest of \$327.86, for 1 year and 6 months, at 6 per cent. per annum?

EXPLANATION. 1 year and 6 months=18 months, and 18÷2=9, the required per cent. Therefore,

\$ 3 2 7.8 6
.0 9
\$ 2 9.5 0 7 4=\$29.507+, *Ans.*

9 per cent. is 9 hundredths, and properly expressed, .09. (§ **85**.)

2. What is the interest of \$256.81, for 2 years, 3 months, and 12 days, at 6 per cent. per annum?

2 years and 3 months=27 months=13 cents and 5 mills, or .135, the per centage for 2 years and 3 months; and 12 days÷6=.002, the per centage for 12 days; and .135+.002=.137, is the per centage for the whole time.

Therefore,

2 5 6.8 1
.1 3 7
1 7 9 7 6 7
7 7 0 4 3
2 5 6 8 1
3 5.1 8 2 9 7.

Hence, \$35.18+, is the interest required.

3. Required the interest of \$196.18, for 1 year, 10 months, and 18 days, at 6 per cent. *Ans.* \$22.168+.

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138. What is the rule for casting interest?

4. Required the interest of $3786.97, for 2 years, 7 months, and 21 days, at 6 per cent. *Ans.* $600.234+.

5. Required the interest of $821.25, for 4 years, 6 months, and 24 days, at 6 per cent. *Ans.* $225.022+.

6. Required the interest of $87.90, for 3 years, 11 months, and 8 days, at 6 per cent. *Ans.* $20.773+.

§ **139.** Whenever interest at any other than 6 per cent. is required, *first find the interest at* 6 *per cent. as above, then divide the interest so found by* 6, *and multiply the quotient by the required per cent.*

The reason for this is obvious. Dividing the interest of any sum of money at 6 per cent. by 6, *gives* 1 *per cent on the same money*, and this multiplied by the given per cent. *gives the interest required.*

7. What is the interest of $358.50, at 8 per cent., for 2 years, 3 months, and 12 days?

EXPLANATION. $358.50×.137=$49.114+=interest at 6 per cent; and $49.114÷6=$8.185+=interest at 1 per cent.; and $8.185×8=$65.480, *Ans.*

8. What is the interest of $150.67, for 2 years, 6 months, and 15 days, at 7 per cent.? *Ans.* $26.806+.

9. What is the interest of $280.75, for 3 years, 1 month, and 27 days, at 9 per cent.? *Ans.* $79.803+.

10. What is the interest of $456.12, for 1 year, 8 months, and 19 days, at 5 per cent.? *Ans.* $39.21+.

§ **140.** When the interest of any sum of money, for a specified time, is ascertained, *the amount due is found by adding the interest to the principal.*

11. What is the amount of $382.16, for 5 years, 6 months, and 18 days, at 6 per cent.?

The interest on the above sum, for the time and rate specified, is $127.259; and $127.259, interest+$382.16, principal=$509.419, amount.

12. What is the amount of $636,93, for 2 years, 7 months, and 28 days, at 6 per cent? *Ans.* $738.626+.

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139. How proceed when the interest is some other than 6 per cent? What is the reason for it?

140. *How is* the amount found?

13. What is the amount of $57.34, for 1 year, 4 months, and 27 days, at 6 per cent.? *Ans.* $62.185+.

14. What is the amount of $288.25, for 2 years, 2 months, and 14 days, at 8 per cent.? *Ans.* $339.11.

In the following sums, the scholar will carefully observe when the *amount*, and when the *interest* only, is required.

15. What is the interest of $106.75, for 1 year, 11 months, and 6 days, at 6 per cent.? *Ans.* $12.383.

16. What is the amount of $731.82, for 3 years, 2 months, and 9 days, at 7 per cent.? *Ans.* $895.32.

17. What is the amount of $1693, for 2 years, 11 months, and 15 days, at 6 per cent.? *Ans.* $1993.507.

18. What is the interest of $860, for 4 years and 6 months, at 8 per cent.? *Ans.* $309.60.

19. What is the interest of $789.32, for 3 years and 8 months, at 6 per cent.? *Ans.* $173.65.

20. What is the interest of $654.18, for 1 year, 2 months, and 8 days, at 6 per cent.? *Ans.* $46.664+.

21. What is the amount of $539.72, for 2 years, 3 months, and 15 days, at 4 per cent.? *Ans.* $589.194+.

22. What is the amount of $18.63, for 2 years and 2 months, at 3 per cent.? *Ans.* $19.84+.

23. What is the amount of $101.73, for 9 months and 12 days, at 6 per cent.? *Ans.* $106.51+.

24. What is the interest of $66.66, for 1 year, 9 months, and 18 days, at 6 per cent.? *Ans.* $7.199+.

25. What is the amount of $288.40, for 2 years, 10 months, and 6 days, at 7 per cent.? *Ans.* $345.935+.

TO COMPUTE INTEREST FOR DAYS ONLY.

§ **141.** Interest at 6 per cent. per annum is $\frac{1}{2}$ cent per month, or 1 mill for 6 days, allowing 30 days to each month, since $\frac{1}{2}$ cent$=5$ mills$=.005$, and $30\div 5=6$, the days required for $1 to gain 1 mill. Hence, $\frac{1}{6}$ *part of the days* during which any sum of money is on interest, will be *the interest* due on each dollar of principal in mills. Therefore,

RULE.

Multiply the principal by 1 sixth part of the days during which the money draws interest. If the principal consist of dollars only, the product will be mills, (§ **91** ;) *if it consist of dollars and cents, it will consist of hundredths of mills, and must be reduced to dollars.*

OBS. If preferred, at any time, the scholar *may multiply by the whole number of days, and divide the product by* 6, *with the same result.*

26. What is the interest of $426, for 18 days, at 6 per cent.?

OPERATION. 18 days ÷ 6 = 3, the number of mills of interest each dollar draws during 18 days; therefore, 426 × 3 = 1278 mills = $1.278, *Ans.*

27. What is the interest of $3672, for 69 days, at 6 per cent.? *Ans.* $42.228.

28. What is the interest of $2087.96, for 42 days, at 6 per cent.? *Ans.* $14.615+.

29. What is the amount of $59.40, for 90 days, at 6 per cent.? *Ans.* $60.29+.

30. What is the amount of $257.93, for 45 days, at 6 per cent.? *Ans.* $259.864+.

31. What is the interest of $88.52, for 36 days, at 6 per cent.? *Ans.* $0.531+.

32. What is the interest of $69.56, for 32 days, at 8 per cent.? *Ans.* $0.494+.

33. What is the amount of $288.75, for 18 days, at 4 per cent.? *Ans.* $289.327+.

34. What is the amount of $375, for 27 days, at 6 per cent.? *Ans.* $376.687+.

35. What is the amount of $480, for 40 days, at 6 per cent.? *Ans.* $483.20.

36. What is the interest of $356, for 63 days, at 6 per cent.? *Ans.* $3.738.

37. What is the interest of $596, for 18 days, at 7 per cent.? *Ans.* $2.086.

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141. What is the rule for casting interest for days only? What is the reason for it? What may be done if preferred?

NUMBERS AND THEIR RELATIONS TO EACH OTHER.

§ 142. Numbers, when we regard their numerical value only, bear a certain relation to each other. Of any two given numbers, the smaller one may be so repeated, that the amount of these repetitions shall just equal the larger, or, such a portion of the larger may be taken as shall just equal the smaller. The relative value of numbers thus ascertained, *is called their ratio.*

It is obvious that numbers, to be thus compared, *must be considered merely as numbers*, representing no particular objects; or, if representing objects of a particular kind, *they must agree in kind.* An obvious and important relation represented by numbers, is that of *quantity and value.* By this we mean, that *the greater the quantity* of any given article, *the greater its value*, and *the less the quantity, the less the value.* To represent quantities and their relative value is then an appropriate use of figures. Thus, if the price of a single yard of cloth be $1, the price or value of 2 yards is $2; of 3 yards, $3; of 4 yards, $4, &c.; that is, *in whatever ratio the quantity increases, the value increases in the same ratio.* Hence, having the value of a single object, *we readily determine the value of any like objects, by repeating that value as many times as there are units in the number representing the objects.*

1. If 1 yard of cloth cost $3, what will 3 yards cost? *Ans.* 3 times $3, that is, $9. What will 4 yards cost? *Ans.* $12. What will 6 yards cost? *Ans.* $18.

2. If the price of 1 pound of butter be 12 cents, what is

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142. What is said of the relation of numbers? What of the smaller of two numbers? What is the relative value of numbers called? How must numbers, to be compared thus, be considered? If they represent objects of a particular kind, what is said of each? What is another important relation represented by numbers? What is meant by this? What is the appropriate design of figures? In what ratio does the value of any quantity increase? Having the value of a single object, how do we determine the value of any number of like objects?

the price of 2 pounds? of 3 pounds? of 5 pounds? of 7 pounds? of 9 pounds?

3. If the price of 1 pound of lard be 6 cents, what is the price of 5 pounds? of 4 pounds? of 7 pounds? of 9 pounds?

4. If a pound of figs cost 8 cents, what will 4 pounds cost? 7 pounds? 5 pounds? 9 pounds? 11 pounds?

§ **143.** If we have the value of one of the equal parts into which any object is divided, the value of the whole is found *by multiplying the value of that part by the number of parts.*

1. If $\frac{1}{4}$ of a ton of hay cost \$4, what will a ton cost? *Ans.* 4 fourths make a whole one, therefore, \$4×4=\$16, the price of a ton.

2. If $\frac{1}{8}$ of an acre of land cost \$12, what is a whole acre worth? *Ans.* \$96.

3. If $\frac{1}{6}$ of a hogshead of molasses cost \$7, what will a hogshead cost? *Ans.* \$42.

4. If $\frac{1}{5}$ of a farm be valued at \$2375, what is the whole farm worth? *Ans.* \$7125.

§ **144.** The value of a number of objects being given, we find the value of one, *by dividing that value by the number of objects;* also, the value of any number of objects, *by multiplying the value of one by the number of objects.*

Whatever be the ratio between the number of objects whose value is given, and of those whose value is required, the same ratio must exist between *the given value* and *the required value.*

This relation of numbers is called *Proportion.*

1. If 3 yards of cloth cost \$12, how many dollars will 9 yards cost?

EXPLANATION. 9 yards are 3 times 3 yards, and will consequently cost 3 times as much money, that is 3 times \$12; and 3 times \$12=\$36, *Ans.*

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143. If we have the value of one of the equal parts of an object, how do we find the value of the whole?

144. Having the value of any number of objects, how do we find the value of one? How find the value of any other number? What ratio must the given value bear to the required value? What is this relation of numbers called?

2. If 4 yards of ribbon cost 20 cents, what will 8 yards cost? *Ans.* 40 cents.

OBS. 8 yards will cost twice as much as 4 yards.

3. If 3 quarts of oil cost 18 cents, what will 18 quarts cost? *Ans.* 108 cents.

4. If 5 yards of cloth cost \$15, what will 20 yards cost? *Ans.* \$60. What will 30 yards cost? *Ans.* \$90. What will 35 yards cost? *Ans.* \$105.

5. If 7 pounds of beef cost 21 cents, what will 21 pounds cost? *Ans.* 63 cents. 28 pounds? *Ans.* 84 cents. 35 pounds? *Ans.* 105 cents.

6. If 6 pounds of raisins cost 24 cents, what will 18 pounds cost? *Ans.* 72 cents.

§ **145.** If the value of any number of the equal parts into which any object or its value is divided be given, the parts being less than the whole, *whatever be the ratio between the parts and the number of such parts in the integral object, the same is the ratio between the value of the parts given, and of those constituting the integral object.*

1. If $\frac{3}{4}$ of a ton of hay cost \$12, what will a whole ton cost?

EXPLANATION. Since 3 fourths cost \$12, 1 fourth will cost $\frac{1}{3}$ of \$12, or \$4; and 4 fourths, 4 times 1 fourth; hence, $\$4 \times 4 = \$16$, *Ans.*

2. If $\frac{5}{6}$ of a man's farm consist of 35 acres, how many acres has he? *Ans.* 42 acres.

3. If $\frac{3}{8}$ of a barrel of pork cost \$6, what will a barrel cost? *Ans.* \$16.

4. If $\frac{6}{7}$ of a hogshead of molasses cost \$30, what will a hogshead cost? *Ans.* \$35.

5. If $\frac{5}{9}$ of a hundredweight of sugar cost \$10, what will a hundredweight cost? *Ans.* \$18.

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145. Having the value of any of the equal parts of an object, how find the value of the whole?

146. How does the ratio between any two sets of objects and their *value compare?*

§ **146.** From the examples here given it is obvious, that, *whatever be the ratio between* any two objects of the same kind, *the same ratio exists between their value.*

§ **147.** Hence it is obvious, that whenever *the value of a certain number of objects or parts of objects is given, and that of a certain other number is required, the objects or parts of objects, of which the value is given, will form a divisor, and the given value, a dividend; while the number of objects or parts of objects, whose value is required, will become the multiplier.*

1. If 9 yards of cloth cost \$27, what will 15 yards cost?

PERFORMED. \$27÷9=\$3, price of one yard, and \$3×15=\$45, the price of 15 yards.

The *principle of cancellation may obviously be applied,* in the solution of sums like the above. (§ **97.**)

RULE.

Analyze the sum, so as to determine what numbers are divisors and what are multipliers; then, placing the number to be operated upon, together with the multipliers, above a horizontal line, and the divisors below, reject the factors common to both, and multiply and divide the uncanceled numbers. (§ **97.**)

Taking the analysis given above, \$27 must be divided by 9 yards and the quotient multiplied by 15 yards. Hence,

$$\frac{\$27.\ 15}{9}. \quad \text{Canceled:} \quad \frac{\overset{3}{\cancel{\$27.}}\ 15}{\cancel{9}}, \text{ and } \$3\times 15=\$45, \textit{ Ans.}$$

2. If 6 pounds of coffee cost 54 cents, what will 14 pounds cost? *Ans.* \$1.26.

3. If 7 tons of hay cost \$70, what will 18 tons cost? *Ans.* \$180.

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147. When the value of a certain number of objects or parts of objects is given, and the value of a certain other number of objects or parts of objects required, what forms the divisor, what the dividend, and what the multiplier? How analyze the sum? What numbers are placed above the line? What numbers below the line? What is done with the equal factors of the upper and lower lines?

4. If 16 men earn $298, what will 35 men earn? *Ans.* $455.

5. If 48 pounds of powder cost $16, what will 39 pounds cost? *Ans.* $13.

6. If 12 gallons of wine are worth $18, what is the value of 22 gallons? *Ans.* $33

7. If 49 barrels of flour cost $245, what will 62 barrels cost? *Ans.* $310.

8. If $\frac{3}{8}$ of a vessel be worth $6270, how much is the whole vessel worth? *Ans.* $16720.

Here we have the value of $\frac{3}{8}$ of the vessel given. Hence, $\frac{6270}{3}$=price of $\frac{1}{8}$; and $\frac{6270.\ 8}{3}$,=price of the whole.

9. If $\frac{5}{9}$ of a piece of land be worth $65, what is the value of the whole at the same rate? *Ans.* $117.

10. If 9 pounds of tea cost 45 shillings, what will 13 pounds cost? *Ans.* 65*s.*

11. If 16 hundredweight of beef cost $96, what will 21 hundredweight cost? *Ans.* $126.

§ **148.** When the *upper numbers only* contain decimals, the *decimals in the answer must equal those given;* but when *the upper and lower numbers both contain decimals,* the decimals in the answer must equal *the excess of those in the upper, over those in the lower numbers.* (§ **89.**)

1. Bought 16 boxes of raisins for $24.32, what would 22 boxes cost? *Ans.* $33.44.

2. Bought 75 yards of cloth for $112.50, what is the value of 84 yards? *Ans.* $126.

§ **149.** If a mixed number (§ **105**) expresses the number of articles whose price is given, the price of a single article is found, *by changing the mixed number to an improper fraction,* (§ **114,**) *and inverting the same* in the statement.

If the articles, *whose price is required,* are expressed by a *mixed* number, *convert that number to an improper frac-*

148. What is the rule for decimals in cancellation?

149. How is the price of a single article found, when the price of a *mixed number* is given?

tion and place the numerator above, and the denominator below the line, as in the following example:

1. Bought 12 pounds of honey for 72 cents; required the value of $13\frac{1}{2}$ pounds.

Statement: $\frac{72.\ \ 27}{12.\ \ 2'}=81$ cents. Here $.72\div 12$ pounds= price of 1 pound, and this again divided by 2, gives the price of $\frac{1}{2}$ pound. Now $13\frac{1}{2}$, whose value is required, $=\frac{27}{2}$ of a pound. Hence, the price of 1 half pound multiplied by 27=the cost or value of $13\frac{1}{2}$ pounds.

2. If 8 loads of hay be valued at \$100, what is the value of $18\frac{3}{4}$ loads? *Ans.* \$230.

3. Bought $7\frac{1}{3}$ barrels of molasses, for \$44; required the value of 27 barrels. *Ans.* \$162.

4. If 15 hundredweight of beef cost \$80, what will $17\frac{1}{3}$ hundredweight cost? *Ans.* \$92.444+.

5. Paid $\$37\frac{1}{2}$ for 15 loads of wood; required the value of 29 loads. *Ans.* \$72.50.

6. Paid $\$18\frac{3}{4}$ for 10 casks of soap; required the value of 32 casks. *Ans.* \$60.

7. What is the value of a man's farm, if $\frac{3}{16}$ of the same be worth \$2700? *Ans.* \$14400.

§ **150.** It often occurs in business computations, *that a result produced by two combined agencies is given, while a similar result of two other agencies, the same in kind with the former, is required.* This constitutes, what in arithmetic is called COMPOUND PROPORTION.

The following example will serve as an illustration:

1. If 3 men, in 5 days, earn \$45, what will 7 men earn in 9 days?

Here the two agencies, *whose result is given,* are 3 men and 5 days, and the result of their combined agency, \$45. The two agencies of the same name, *whose result is re-*

How is the price of a number of articles found, the value of one being given?

150. What often occurs in business? What is this called in ordinary arithmetic?

quired, are 7 men and 9 days. But $45 ÷ 3 men = the dollars 1 man earns in 5 days, and this value divided by the 5 days = what 1 man earns in 1 day. The 3 and the 5, therefore, *become divisors in the natural analysis of the sum.* Again, if what 1 man earns in 1 day be multiplied by 7, the result will be what 7 men earn in 1 day, and this multiplied by 9, shows what 7 men will earn in 9 days. In the analysis of the work then, the 7 and 9 become *multipliers.*

RULE.

Divide the given value by the agencies which produce it, and multiply the quotient by the agencies, the result of whose combined action is required.

Statement of preceding sum: $\frac{\$45.\ 7.\ 9}{3.\ 5} = \$189$, *Ans.*

2. If it require 7 horses, 16 days to remove 104 tons of hay a certain distance, how many tons will 14 horses remove in 21 days? *Ans.* 273 tons

Statement: $\frac{104.\ 14.\ 21}{7.\ 16} = 273.$

3. If 16 men earn $192 in 8 days, what will 12 men earn in 27 days? *Ans.* $486.

4. If 8 men build 18 rods of wall in 12 days, how many rods will 96 men build in 2 days? *Ans.* 36 rods.

5. If 8 men in 15 days consume 48 pounds of beef, how many pounds will 20 men consume in 25 days?

Ans. 200 pounds.

6. If 9 horses eat 128 bushels of oats in 64 days, how many bushels will 24 horses eat in 32 days?

Ans. 170⅔ bushels.

7. If 12 men reap 36 acres of wheat in 6 days, how many acres will 18 men reap in 9 days? *Ans.* 81 acres.

Sum 2. What does dividing by 7 and 9 show? What is the rule? Sum 3. Dividing $192 by 16 shows what? What does dividing that result by 8 show? Multiplying what 1 man earns in 1 day by 12, *shows what?* That result multiplied by 27 shows what? Similar *analytical* questions should be asked respecting each sum.

8. If 15 men in 21 days spend $472.50, how much will 27 men spend in 35 days? *Ans.* $1417.50.

9. If $300 in 5 years gain $90 interest, what will $450 gain in 6½ years? *Ans.* $175.50.

10. If 5 men in 12 days make 180 pairs of shoes, how many pairs will 15 men make in 4 days? *Ans.* 180 pairs.

11. If 7 men in 9 days cut 84 cords of wood, how many cords will 12 men cut in 21 days? *Ans.* 336 cords.

12. If 25 persons in 3 years consume 300 bushels of wheat, how many bushels will 27 persons consume in 13 years? *Ans.* 1404 bushels.

13. If it require 16 men, 12 days to make 64 suits of clothes, how many suits can be made by 24 men in 15 days? *Ans.* 120 suits.

§ **151.** A result produced by the united action of two given agents; also a result *to be produced*, and *one* of the agents employed, are often given, and a second agent is required, which acting unitedly with that given, shall secure the required result.

The following example will serve as an illustration:—

1. If 8 men build 18 rods of wall in 12 days, how many men will build 36 rods in 2 days?

A number of men is required, who, operating 2 days, shall build 36 rods of wall; but 36 rods will require more men than 18 rods. Multiply, therefore, 8 men by 36 and divide the product by 18. Also 2 days, the time in which this greater work is to be performed, is *less* than the 12 days in which the other work was performed. More men will, therefore, be required. Hence, when one of two agencies is required, an increase in the *required result* requires a *corresponding increase* of that agency; but an increase in one of the two agencies necessarily requires *a corresponding decrease in the other*. Therefore,

To obtain *an increase in a required agency, we multiply that agency by the result to be attained, and also by the*

151. How is a second agent found, when required? Give the analysis of the sum. When two agencies are employed, what is requisite, when an increased result is to be obtained? When one agency is increased, how is the other affected?

greater of the two given agencies corresponding in kind, and divide by the attained result and the less of the two given agencies; but to obtain a less agency, we multiply by the less of the given agencies and divide by the greater, the terms expressing the results being used as before.

The statement for the previous sum is therefore,

$\frac{8.\ 36.\ 12}{18.\ 2}$=96 men, *Ans.* Here the number of men required is increased, both by the increase of work and the decrease of the time.

Obs. The principle involved in the above is simply, that *with a fixed result to be attained, as one of the agencies employed is increased, the other is diminished.*

2. If 5 men make 120 pairs of shoes in 12 days, how many men will make 150 pairs in 15 days?

Statement: $\frac{5.\ 150.\ 12}{120.\ 15}$=5 men, *Ans.*

3. If 4 men in 12 days earn $96, in how many days will 8 men earn $288? *Ans.* 18 days.

4. If 12 men reap 120 acres of wheat in 10 days, in how many days will 30 men reap 200 acres? *Ans.* 6⅔ days.

COMPOUND NUMBERS.

§ 152. *Compound numbers increase in no uniform ratio.*

In Avoirdupois weight, 16 ounces are required for a pound; and in Troy and Apothecaries' weight, 12 ounces make a pound. So in time, the denominations are seconds, minutes, hours, days, weeks, &c. But none of these denominations sustain to each other a uniform relation, since it requires 60

How do we obtain an increased agency? How a less agency? What is the principle involved?

152. How do compound numbers increase? What is said of *compound numbers?*

seconds to make 1 minute, 60 minutes to make 1 hour, 24 hours to make 1 day, &c.

Compound Numbers do not, however, differ in *kind*, for a lower denomination repeated a required number of times, will produce a higher denomination. Hence the lower are but parts of the higher denominations. They are, however, compound in their relations to each other; that is, their relative numerical value is constantly changing, being expressed at one time by the numbers, 4, 12, and 20, as in English money; at another by 3, 12, 3, 5½, 40, 8, &c., as in Long measure.

TABLES OF COMPOUND NUMBERS.

TABLE I.—ENGLISH MONEY.

4 farthings (*qr.*) . .	*make*	1 penny, . . .	*marked*	*d.*
12 pence	"	1 shilling, . . .	"	*s.*
20 shillings . . .	"	1 pound, . . .	"	£.

The pound and farthing are denominations of English money *not coined*. The English sovereign, a common English coin, is of the same value as the pound sterling.

TABLE II.—TROY WEIGHT.

24 grains (gr.) . .	*make*	1 pennyweight .	*marked*	pwt.
20 pennyweights .	"	1 ounce, . . .	"	oz.
12 ounces . . .	"	1 pound, . . .	"	lb.

By this weight, jewels, precious metals, and distilled liquors are weighed. The pound Troy is the same as the Imperial pound Troy of England, requiring 5760 grains to make an ounce.

TABLE III.—APOTHECARIES' WEIGHT.

20 grains (gr.) . .	*make*	1 scruple,	*marked*	℈. or ʒ.
3 scruples . . .	"	1 dram, .	"	dr. or ʒ.
8 drams . . .	"	1 ounce, .	"	oz. or ℥.
12 ounces . . .	"	1 pound, .	"	℔.

This weight is employed by apothecaries and physicians, in weighing medicines. The pound and ounce are the same as in Troy weight. The drams, scruples, and grains, are minute divisions of weight, to enable the physician to weigh out small doses of medicine.

What are such numbers called? In what do they not differ? Why? In what respect are they compound?

TABLE IV.—AVOIRDUPOIS WEIGHT.

16 drams (dr.)	*make*	1 ounce,	*marked*	oz.
16 ounces	"	1 pound,	"	lb.
25 pounds	"	1 quarter,	"	qr.
4 quarters	"	1 hundredweight,	"	cwt.
20 hundredweight	"	1 ton,	"	T.

This is the weight used by grocers, and is employed for weighing all coarse articles: as sugar, tea, coffee, cheese, butter, hay, &c. The pound Avoirdupois is heavier than the pound Troy. The pound Troy=5760 grains, while the pound Avoirdupois=7000 grains.

TABLE V.—LONG MEASURE.

3 barley corns (b. c.)	*make*	1 inch,	*marked*	in.
12 inches	"	1 foot,	"	ft.
3 feet	"	1 yard,	"	yd.
5½ yards, or 16½ feet,	"	1 rod,	"	rd.
40 rods	"	1 furlong,	"	fur.
8 furlongs	"	1 mile,	"	m.
3 miles	"	1 league,	"	l.
60 geographic, or 69½ statute miles,	"	1 degree,	"	°.

360° make a circle. That circle, 1 degree of which is 60 geographic or 69½ statute miles, is the circumference of the earth.—This measure is used for measuring length, and is often called *linear measure.*

TABLE VI.—CLOTH MEASURE.

2¼ inches (in.)	*make*	1 nail,	*marked*	na.
4 nails, or 9 inches,	"	1 quarter,	"	qr.
4 quarters	"	1 yard,	"	yd.
3 quarters	"	1 ell Flemish,	"	e Fl.
5 quarters	"	1 ell English,	"	e. E.
6 quarters	"	1 ell French,	"	e. Fr.

The yard in this measure is the same as in Long measure, viz.: 3 *ft. or 36 inches.* Hence, 36÷4=9 in., the quarter; and 9÷4=2¼ in., *the nail.*

TABLE VII.—SQUARE MEASURE.

144 square inches	*make*	1 square foot,	*marked*	sq. ft.
9 square feet	"	1 square yard,	"	sq. yd.
30¼ square yards, or 272¼ square feet,	" "	1 square rod,	"	sq. rd.
40 square rods	"	1 rood,	"	R.
4 roods (160 sq. rods)	"	1 acre,	"	A.
640 acres	"	1 square mile,	"	sq. m.

This measure is employed in measuring surfaces, or that which has length and breadth, as flooring, plastering, paving, land, &c.

TABLE VIII.—CUBIC MEASURE.

1728 cubic inches	*make*	1 cubic foot,	*marked*	cu. ft.
27 cubic feet	"	1 cubic yard,	"	cu. yd.
40 feet of hewn, or 50 feet of round timber,	"	1 ton,	"	T.
16 cubic feet	"	1 cord foot,	"	c. ft.
8 cord feet, or 128 cubic feet,	"	1 cord,	"	C.

This measure is employed for measuring solid bodies, that is, bodies having length, breadth, and thickness; as boxes, bins, timber, stone, &c. The cord of wood contains 128 solid feet. Hence, a pile of wood 8 feet long, 4 feet high, and 4 feet thick, contains just a cord, since $8 \times 4 \times 4 = 128$.

TABLE IX.—WINE MEASURE.

4 gills (gi.)	*make*	1 pint,	*marked*	pt.
2 pints	"	1 quart,	"	qt.
4 quarts	"	1 gallon,	"	gal.
31½ gallons	"	1 barrel,	"	bar.
42 gallons	"	1 tierce,	"	tier.
63 gallons	"	1 hogshead,	"	hhd.
2 hogsheads	"	1 pipe,	"	P.
2 pipes	"	1 tun,	"	T.

This measure is employed to measure wine, alcohol, oil, and all other liquors, except beer, ale, and milk. The gallon measure contains 231 cubic inches. A gallon of pure water, at the temperature of 41 degrees, which is nearly its greatest density, weighs $8\frac{339}{1000}$ pounds avoirdupois. The British Imperial gallon contains 277.274 cubic inches.

TABLE X.—BEER MEASURE.

2 pints (pt.)	. .	*make*	1 quart,	*marked*	qt.
4 quarts	. . .	"	1 gallon,	"	gal.
36 gallons	. . .	"	1 barrel,	"	bar.
54 gallons	. . .	"	1 hogshead,	"	hhd.

This measure is the appropriate one for measuring beer, milk, and ale, but has, for the most part, fallen into disuse, and wine measure taken its place. The beer gallon contains 282 cubic inches.

TABLE XI.—DRY MEASURE.

2 pints (pt.)	. .	*make*	1 quart,	*marked*	qt.
8 quarts	. . .	"	1 peck,	"	pk.
4 pecks	. . .	"	1 bushel,	"	bu.
36 bushels	. . .	"	1 chaldron,	"	ch.

This measure is employed for measuring all kinds of grain, fruit, salt, &c. The standard of measure is the Winchester bushel, which contains 2150.4 cubic inches.

TABLE XII.—TIME.

60 seconds (sec.)	.	*make*	1 minute,	*marked*	m.
60 minutes	. . .	"	1 hour,	"	h.
24 hours	. . .	"	1 day,	"	d.
7 days	. . .	"	1 week,	"	w.
4 weeks	. . .	"	1 month,	"	mo.
12 months, or 365 days, 6 hours,		"	1 year,	"	yr.

The 6 hours over 365 days in each year, amount to 24 hours, or 1 day, in 4 years. Hence, every fourth year contains 366 days. This additional day is joined to the month of February. The following lines will aid the scholar in remembering the number of days in each month.

"Thirty days hath September,
April, June, and November;
February twenty-eight alone;
All the rest have thirty-one,
Except in leap-year, then's the time
When February has twenty-nine."

TABLE XIII.—CIRCULAR MEASURE.

60 seconds (″) . .	*make*	1 minute, . .	*marked* ′.
60 minutes . . .	"	1 degree, . .	" °.
30 degrees . . .	"	1 sign, . . .	" S.
12 signs, or 360°, .	"	1 circle, . . .	" cir.

This measure marks the divisions of the circle. It is employed in measuring latitude and longitude, also in measuring the motion of the heavenly bodies.

TABLE OF PARTICULARS.

12 things	*make*	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 single objects . . .	"	1 score.
56 pounds of butter . .	"	1 firkin.
112 pounds	"	1 quintal of fish.
196 pounds	"	1 barrel of flour.
200 pounds	"	1 barrel of pork.
14 pounds of lead . . .	"	1 stone.
2½ stones	"	1 pig.
8 pigs	"	1 fother.
24 sheets of paper . .	"	1 quire.
20 quires of paper . .	"	1 ream.

N. B.—When a sheet of paper is folded so as to produce 18 leaves, it is called . . . . . 18mo.
When into 12 leaves, it is called . Duodecimo, or 12mo.
When into 8 leaves, " " . . . Octavo, or 8vo.
When into 4 leaves, " " . . . Quarto, or 4to.
When into 2 leaves, " " . . . Folio.

§ **153.** The denominations composing each of the preceding Tables *agree in kind*, but *differ in value;* the higher denominations being, in each case, produced by the repetitions of the lower denominations. The number of units required of a lower denomination to make 1 of the next higher denomination is, however, constantly varying. Hence, in operating with Compound Numbers, we have no uniform number by which we divide in reducing the lower to the higher denominations, or by which we multiply in bringing high denominations to those of lower value, but must in each case be guided entirely by the numerical relation the denomination operatod upon, sustains to the other kindred denominations.

COMPOUND ADDITION.

RULE.

§ 154.—1. *Write down the several numbers, placing the same denominations under each other.*

2. *Beginning with the lowest denomination, add each column separately; and dividing the amount of each by the number required of the denomination added to make one of the next higher denomination, write the remainder at the foot of the column, and add the quotient to the next column.*

3. *Continue this process through all the denominations, and write down the whole amount of the last or left-hand column.*

1. What is the amount of £33 8*s.* 9*d.* 2*qr.*; £16 12*s.* 3*d.* 1*qr*; £23 13*s.* 11*d.* 3*qr.*; and £37 10*s.* 7*d.* 2*qr.*?

OPERATION.

£.	*s.*	*d.*	*qr.*
33	8	9	2
16	12	3	1
23	13	11	3
37	10	7	2
111	5	8	0

It will be observed that pounds are written under pounds, shillings under shillings, pence under pence, and farthings under farthings.

We first add the farthings, and obtain 8 as the amount = 2*d.* and no remainder. We therefore write a cipher under farthings, and add the 2 pence into the column of pence, and find the amount to be 32*d.* = 2*s.* 8*d.* We therefore write the 8*d.* at the foot of the column, and add the 2*s.* into the column of shillings, and find the amount to be 45*s.* = £2 5*s.*; and writing down the 5*s.* we add the £2 into the column of pounds, and find the amount of that column to be £111, and there being no denomination higher than pounds, we write down the whole amount, viz., £111. The answer to the sum is therefore, £111 5*s.* 8*d.* 0*qr.* (Table I.)

154. In compound addition how are the numbers to be written? With what denomination do we begin to add? How add? By what do you divide the amount of each column? Where do you write the *remainder*? To what add the quotient? What is written down at *the left-hand* column?

We prove an operation in Compound Addition in the same manner as in Simple Addition. (§ 32.)

2. What is the amount of £13 17*s.* 11*d.* 1*qr.*; £22 14*s.* 9*d.* 1*qr.*; £37 18*s.* 6*d.* 3*qr.*; and £46 13*s.* 7*d.* 2*qr.*?
Ans. £121 4*s.* 10*d.* 3*qr.*

3. What is the amount of £75 19*s.* 11*d.* 3*qr.*; £63 17*s.* 5*d.* 1*qr.*; £41 12*s.* 3*d.* 3*qr.*; and £11 15*s.* 8*d.* 3*qr.*?
Ans. £193 5*s.* 5*d.* 2*qr.*

4. What is the amount of £105 1*s.* 2*d.* 3*qr.*; £218 11*s.* 5*d.* 2*qr.*; £199 17*s.* 9*d.* 2*qr.*; and £77 18*s.* 3*d.* 3*qr.*?
Ans. £601 8*s.* 9*d.* 2*qr.*

5. What is the amount of £1 11*s.* 9*d.* 1*qr.*; £7 7*s.* 7*d.* 3*qr.*; £17 13*s.* 9*d.* 3*qr.*; and £21 14*s.* 6*d.* 1*qr.*?
Ans. £48 7*s.* 9*d.* 0*qr.*

6. A silversmith bought of A, 3 lb. 9 oz. 14 pwt. 16 gr. of silver; of B, 9 lb. 11 oz. 17 pwt. 18 gr.; of C, 1 lb. 8 oz. 19 pwt. 21 gr.; and of D, 3 lb. 7 oz. 12 pwt. 16 gr. How much silver did he buy? (Table II.)
Ans. 19 lb. 2 oz. 4 pwt. 23 gr.

7. What is the amount of 16 ℔ 8 ℥ 3 ℨ 1 ℈ 8 gr.; 8 ℔ 8 ℥ 1 ℨ 2 ℈ 5 gr.; 9 ℔ 10 ℥ 4 ℨ 2 ℈; and 21 ℔ 3 ℥ 7 ℨ 1 ℈ 1 gr.? (Table III.)
Ans. 56 ℔ 7 ℥ 1 ℨ 0 ℈ 14 gr.

8. What is the amount of 15 cwt. 3 qr. 24 lb. 8 oz. 13 dr.; 21 cwt. 1 qr. 16 lb. 13 oz. 6 dr.; 87 cwt. 2 qr. 12 lb. 3 oz. 15 dr.; and 27 cwt. 1 qr. 17 lb. 13 oz. 9 dr.? (Table IV.) *Ans.* 102 cwt. 1 qr. 21 lb. 7 oz. 11 dr.

9. What is the amount of 12 yd. 3 qr. 3 na.; 17 yd. 1 qr. 2 na.; 81 yd. 3 qr. 3 na.; and 5 yd. 2 qr. 1 na.? (Table V.) *Ans.* 117 yd. 3 qr. 1 na.

10. What is the amount of 16 m. 5 fur. 27 rd. 3 yd. 2 ft. 9 in. 1 b. c.; 12 m. 7 fur. 31 rd. 4 yd. 1 ft. 10 in. 2 b. c.; 21 m. 3 fur. 12 rd. 1 yd. 1 ft. 11 in. 2 b. c.; and 25 m. 2 fur. 15 rd. 4 yd. 2 ft. 3 in. 1 b. c.? (Table VI.)
Ans. 76 m. 3 fur. 7 rd. 3 yd 2 ft. 11 in.

11. What is the amount of 16 hhd. 41 gal. 1 qt 1 pt.; 21 hhd. 17 gal. 3 qt.; 63 hhd. 2 qt. 1 pt.; 36 hhd. 45 gal. 1 pt.? (Table X.) *Ans.* 137 hhd. 41 gal. 3 qt. 1 pt.

How do we prove simple addition?

12. What is the amount of 27 sq m. 32 A. 1 R. 15 sq. rd. 18 sq. yd. 5 sq. ft. 11 sq. in.; 13 sq. m. 46 A. 3 R. 19 sq. rd. 29 sq. yd. 8 sq. ft. 9 sq. in.; and 81 sq. m. 56 A. 2 R. 27 sq. rd. 6 sq. ft. 3 sq. in.? (Table VII.)

Ans. 121 sq. m. 135 A. 3 R. 22 rd. 18¾ yd. 1 ft. 23 in.

13. What is the amount of 99 c. ft. 420 c. in.; 78 c. ft. 864 c. in.; 320 c. ft. 740 c. in.; 950 c. ft. 222 c. in.; and 48 c. ft. 12 c. in.? (Table VIII.)

Ans. 11 C. 88 c. ft. 530 c. in.

14. What is the amount of 7 hhd. 41 gal. 3 qt. 1 pt. 3 gi.; 5 hhd. 53 gal. 1 qt. 1 pt. 1 gi.; 12 hhd. 5 gal. 1 pt. 3 gi.; and 17 hhd. 17 gal. 3 qt. 1 gi.? (Table IX.)

Ans. 42 hhd. 55 gal. 1 qt. 1 pt.

15. What is the amount of 6 ch. 27 bu. 3 pk. 7 qt. 1 pt.; 9 ch. 18 bu. 3 qt; 28 bu. 3 pk. 5 qt.; and 18 bu. 1 pk. 3 qt. 1 pt.? (Table XI.) *Ans.* 17 ch. 21 bu. 1 pk. 3 qt.

16. What is the amount of 2 yr. 9 mo. 27 d. 13 h. 22 m. 56 sec.; 8 yr. 11 mo. 3 d. 21 h. 43 m. 21 sec.; 1 yr. 11 mo. 18 d. 23 h. 59 m. 52 sec.; and 2 yr. 8 mo. 26 d. 17 h. 36 m. 8 sec.? (Table XII.)

Ans. 16 yr. 5 mo. 17 d. 4 h. 42 m. 17 sec.

COMPOUND SUBTRACTION.

§ **155.** Compound Subtraction consists in taking one compound number from another.

1. From £16 7*s.* 8*d.* 3*qr.*, take £13 9*s.* 10*d.* 2*qr.*

	£	*s.*	*d.*	*qr.*
OPERATION.	16	7	8	3
	13	9	10	1
	2	17	10	2

We here say, 1 *qr.* taken from 3 *qr.*=2 *qr.*, which we write down. But 10*d.* cannot be taken from 8*d.* Therefore 1*s.*=12*d.*+8*d.*=20*d.* and 20*d.*−10*d.*=10*d.*, which we write down, and carry 1 to the lower figure in the column of shillings, which, consequently, becomes 10*s.*, and 7*s.*+20*s.* borrowed=27*s.*, and 27*s.*−10*s.*=17*s.*, which is also written down. We must carry 1 to the lower figure in the column of *pounds*, which becomes, in consequence, £14, and £16−£14=£2. *Hence, the* answer is, £2 17*s.* 10*d.* 2*qr.*

We hence derive the following

RULE.

1. *Write the subtrahend under the minuend, so that each denomination shall stand under one of the same name, and draw a line beneath them.*

2. *Commencing with the right-hand denomination, if practicable, take each denomination in the subtrahend from the corresponding one in the minuend.*

3. *If, in any instance, this be impracticable, the lower number being the largest, increase the upper number by as many units as will make one of the next higher denomination, subtract the lower number from the number thus produced, and carry* 1 *to the next lower number. This operation continued through all the denominations, will produce the required remainder.*

The proof of the operation is found in the same manner as in simple subtraction, viz.: *by adding the remainder and subtrahend together, their sums will equal the minuend.*

In the following sums, determine the number to be borrowed *by the denomination.*

2. From £8 17*s.* 9*d.* 2*qr.*, take £4 18*s.* 6*d.* 1*qr.*
Ans. £3 19*s.* 3*d.* 1*qr.*

3. From £29 3*s.* 3*d.* 1 *qr.*, take £13 13*s.* 9*d.* 3*qr.*
Ans. £15 9*s.* 5*d.* 2*qr.*

4. From 17 T. 7 cwt. 16 lb., take 12 T. 9 cwt. 3 qr. 18 lb. *Ans.* 4 T. 17 cwt. 23 lb.

5. If 1 silver bowl weigh 2 lb. 9 oz. 12 pwt. 13 gr., and another, 2 lb. 11 oz. 18 pwt. 2 gr., what is the difference in their weight? *Ans.* 2 oz. 5 pwt. 13 gr.

6. If one piece of broadcloth measure 8 yd. 1 qr. 3 na. and another 7 yd, 3 qr. 2 na., what is the difference of their lengths? *Ans.* 2 qr. 1 na.

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155. In what does subtraction consist? In compound subtraction, how do we write the subtrahend? Where do we commence? If the lower figure cannot be taken from that above it, what is done? How many do we carry? How is the work proved?

7. From 25 m. 6 fur. 16 rd. 12 ft. 9 in., take 8 m. 7 fur. 13 rd. 9 ft. 11 in. *Ans.* 16 m. 7 fur. 3 rd. 2 ft. 10 in.

8. From 5 ℔ 9 ℥ 5 3 2 ℈ 16 gr., take 3 ℔ 11 ℥ 6 3 2 ℈ 15 gr. *Ans.* 1 ℔ 9 ℥ 7 3 1 gr.

9. From 21 e. E. 1 qr. 2 na., take 16 e. E. 4 qr. 3 na. *Ans.* 4 e. E. 1 qr. 3 na.

10. From 27 hhd. 19 gal. 3 qt. 1 gi., take 16 hhd. 43 gal. 1 qt. 1 pt. 3 gi. *Ans.* 10 hhd. 39 gal. 1 qt. 2 gi.

11. From 15 m. 4 fur. 27 rd. 4 yd. 2 ft. 11 in. 1 b. c., take 12 m. 3 fur. 36 rd. 3 yd. 1 ft. 9 in. 2 b. c. *Ans.* 3 m. 31 rd. 1 yd. 1 ft. 1 in. 2 b. c.

COMPOUND MULTIPLICATION.

§ **156.** The repetition of a compound number constitutes compound multiplication. The *units* in the multiplier determine the *number of repetitions* the same as in simple multiplication.

1. What is the product of £5 13*s*. 9*d*. 2*qr*. multiplied by 7 ?

	£	*s*.	*d*.	*qr*.
OPERATION.	5	13	9	2
				7
	39	16	6	2 = £39 16*s*. 6*d*. 2*qr*., *Ans.*

Commencing with the 2*qr*., we say, 7 times 2*qr*.=14*qr*.=3*d*. 2*qr*. We write down the 2*qr*. and add the 3*d*. to the product of 9*d*., multiplied by 7, and obtain 66*d*.=5*s*. 6*d*. The 6*d*. we write down, and add the 5*s*. to the product of 13*s*.×7, and obtain 96*s*.=£4 16*s*., and writing down the 16*s*., we add the £4 to the product of £5 multiplied by 7, and obtain £39. This being the highest denomination of English money, we write the whole at the foot of the column of pounds.

RULE.

1. *Write the several denominations of the given compound number in the order of their values, placing the lowest denomination on the right, and directly under that*

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156. What constitutes compound multiplication? How write the *several* denominations? Where place the lowest denomination?

denomination, write the multiplier, and draw a line beneath it.

2. *Multiply the several denominations in the order of their value, and from the product of each carry forward to the product of the next higher denomination, as many units of the higher as can be obtained from the product of the lower denomination, and write the remainder under the denomination of its own name.*

3. *If the last denomination given is the highest of the corresponding table, write the whole amount at the foot of the left-hand column.*

When the multiplier is *a composite number, the component parts of that number may be employed* instead of the number itself. (§ **64**.)

2. Multiply £17 8*s*. 4*d*. 2*qr*. by 8. *Ans*. £139 7*s*.

PERFORMED.

£.	s.	d.	qr.
17	8	4	2
			8
139	7	0	0

Compare this operation with the explanation of the previous sum.

3. Multiply £79 16*s*. 9*d*. 1*qr*. by 12.
Ans. £958 1*s*. 3*d*.

4. Multiply £9 15*s*. 8*d*. by 18, (multiplying by factors.)
Ans. £176 2*s*.

5. What cost 5 cwt. of rice, at £1 7*s*. 9*d*. 2*qr*. per cwt.?
Ans. £6 18*s*. 11*d*. 2*qr*.

6. What cost 8 yards of broadcloth, at £1 2*s*. 3*d*. per yard? *Ans*. £8 18*s*.

7. Suppose the age of each of 27 children to be 6 years, 9 months, 8 days, and 11 hours; how many years, &c., do their united ages amount to, allowing each month to be just 30 days? *Ans*. 182 yr. 10 mo. 18 d. 9 h.

Under what denomination do we place the multiplier? How multiply the several denominations? How many do we carry forward from each product to the next higher denomination? Where write the remainder? How write the product of the highest denomination? How multiply when the multiplier is a composite number?

8. What cost 11 tons of hay, at £2 1s. 10*d.* per ton?
Ans. £23 2*d.*

9. What is the value of 12 bushels of wheat, at 9s. 10*d.* per bushel? *Ans.* £5 18s.

10. What is the value of 27 yards of cloth, at 7s. 6*d.* per yard? *Ans.* £10 2s. 6*d.*

11. What will 32 yards of cloth cost, at 9s. 9*d.* per yard?
Ans. £15 12s.

12. Required the value of 36 gallons of wine, at 5s. 8*d.* per gallon? *Ans.* £10 4s.

§ 157. TO FIND THE VALUE OF A LARGE NUMBER OF ARTICLES, THAT NUMBER NOT BEING COMPOSITE.

RULE.

Take any two numbers whose product most nearly equals the given number or multiplier, without exceeding it, and multiply by each of them successively, retaining their product. Also, multiply the original multiplicand by the number necessary to make up the entire multiplier, and add the product to the preceding product.

1. What is the value of 51 yards, at 3s. 6*d.* per yard?

OPERATION.

	s.	d.	
	3	6	
		7	
£1	4	6	=price of 7 yards.
		7	
8	11	6	=price of 49 yards.
	7	0	=3s. 6*d.*×2=price of 2 yards.
8	18	6	=price of 51 yards.

2. What will 23 gallons of molasses cost, at 3s. 6*d.* per gallon? *Ans.* £4 6*d.*

3. What will 94 yards of cloth cost, at £1 9s. 4*d.* per yard? *Ans.* £137 17s. 4*d.*

4. What will 29 cwt. of sugar cost, at 17s. 8*d.* per cwt.?
Ans. £25 12s. 4*d.*

5. What will 65 cwt. of sugar cost, at 19s. 3*d.* per cwt.?
Ans. £62 11s. 3*d.*

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157. How operate when the multiplier is not composite?

COMPOUND DIVISION.

§ 158. Compound Division consists in resolving a compound number into several equal parts.

1. Divide £17 11*s.* 4*d.* by 8.

	£	*s.*	*d.*
PERFORMED. 8)	17	11	4
	2	3	11

EXPLANATION. £17÷8=2 and 1 remainder. This £1 remainder+11*s.*=31*s.*, and 31*s.*÷8=3 and 7*s.* remainder, and 7*s.*+4*d.*=88*d.*, and 88÷8=11*d.*

It is obvious that the remainder of each division is *of the same denomination as the number divided. The same is true of the quotient* figures.

RULE.

Commence with the highest denomination of the dividend, and if, after dividing, there be a remainder, reduce it to the next lower denomination, and, adding whatever may be given of the lower denomination, divide again; and so continue to do through all the denominations.

If the divisor is a composite number, *the division may be performed by factors, if preferred.*

2. Divide £140 12*s.* 9*d.* by 12. *Ans* £11 14*s.* 4*d.* 3*qr.*
3. Divide £27 16*s.* by 32. *Ans.* 17*s.* 4*d.* 2*qr.*
4. Divide £128 9*s.* by 42. *Ans.* £3 1*s.* 2*d.*
5. Divide 12 cwt. 3 qr. 15 lb. by 10.
Ans. 1 cwt. 1 qr. 4 lb.
6. Divide 5 hhd. 42 gal. 3 qt. by 4.
Ans. 1 hhd. 26 gal. 1 qt. 1 pt. 2 gi.
7. Divide 14 cwt. 1 qr. 12 lb. by 5.
Ans. 2 cwt. 3 qr. 12 lb. 6 oz. 6+ dr.
8. Divide 156 bu. 3 pk. 6 qt. by 18.
Ans. 8 bu. 2 pk. 7 qt.
9. Divide 9 hhd. 28 gal. 2 qt. by 12.
Ans. 49 gal. 2 qt. 1 pt.

158. In what does compound division consist? With what denomination do we commence in dividing? What is done with each remainder? How divide if the divisor is composite?

REDUCTION.

§ **159.** Reduction consists in *changing the denominations of numbers without altering their value.*

Since 1 penny=4 farthings, it is perfectly obvious that any value expressed by pence, is also expressed by a number of farthings, four times as great as the pence.

1. How many farthings are equal to 3 pence? *Ans.* 3×4=12 farthings. How many farthings are equal to 7 pence?

Any value in pence and farthings is expressed in farthings by reducing the pence to farthings and adding the given farthings.

2. How many farthings are there in 5 pence, 2 farthings? $5d.=20qr.$, and $20qr.+2qr.=22qr.$, *Ans.* How many farthings in $9d.$ $3qr.$? In $8d.$ $1qr.$? In $11d.$ $1qr.$?

3. How many pence in $3s.$? Since $1s.=12d.$, $3s.=3$ times $12d.=36d.$, *Ans.* How many pence in $5s.$? in $7s.$?

4. How many pence are there in $4s.$ $3d.$? In $4s.$ there are $48d.$, and $48+3=51d.$, *Ans.*

To find the pence in a given number of shillings and pence, *multiply the shillings by* 12, *and add the given pence.*

5. How many pence in $5s.$ $8d.$? In $4s.$ $7d.$? In $6s.$ $6d.$?

6. How many shillings in £3? In £3 6*s.*? In £2 9*s.*?

§ **160.** To obtain the value of any denomination in terms of higher value, *divide by the number required of the given denomination to make one of the denomination of higher value.*

1. In 16 farthings, how many pence? It requires 4 farthings to make 1 penny; therefore, $16qr.\div 4=4d.$, *Ans.* How many pence in 12 farthings? In 20 farthings?

2. In 36 pence how many shillings? $12d.=1s.$; therefore, $36d.\div 12=3s.$ In 48 pence how many shillings?

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159. In what does reduction consist? How is the value of pence and farthings expressed in farthings? How do we find the pence in any number of shillings and pence?

160. How do we obtain the value of any denomination in higher terms? Of what denomination is the remainder?

If in dividing there be a remainder, it will be of the same kind as the number divided.

3. In 30 pence how many shillings? $30d. \div 12 = 2s.$ and $6d.$ over, *Ans.*

4. In 40 pence how many shillings and pence? In 56d.?

5. In 37s. how many pounds? $37s. \div 20 = £1\ 17s.$, *Ans.* In 43s. how many pounds? In 73s.? In 87s.? In 93s.?

§ **161.** From the preceding examples and remarks, it is obvious that there are two processes of reduction, viz.: One, by which *the value of a higher denomination is fully expressed in terms of a lower value;* and a second, by which *a number is found expressing a given value in terms higher than those of the given denomination.*

The former of these processes, viz.: reducing higher denominations to lower, is called, *Reduction Descending;* while the latter, which consists in bringing low denominations to those of a higher value, is called *Reduction Ascending.*

GENERAL RULES.

1. To reduce high denominations to those that are lower: *multiply the highest of the given denominations by that number which expresses the units required of the denomination next lower to make one of the higher, and to the product, add what is given of the lower denomination. Multiply the number thus obtained by the number requisite to bring it to the next lower denomination, and add whatever is given of that lower denomination; and so continue to do till the required denomination is obtained.*

2. To reduce low denominations to those that are higher: *divide the given denomination by the number required of this denomination to make one of the next higher; divide the number thus obtained, in the same manner; and after each division, carefully observe the remainder.*

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161. How many processes of reduction? How reduce high denominations to their value in lower denominations? What is done when any thing of a lower denomination is given? How proceed? How reduce low denominations to high? Of what denomination will the quotient be in each instance?

The quotient will, in each case, be a higher denomination, and the remainder, if any, will be of the same name as the number divided. This process continued till the required denomination is obtained, will determine the answer.

The former of these rules relates entirely *to bringing high denominations into low;* and the latter, *to bringing low denominations into high.* By neither operation is the *given value affected.*

§ **162.** PROOF. The work in each case is proved by reversing the operation.

1. Reduce £16 to shillings, pence, and farthings.

A *higher* denomination is to be reduced to a lower one. *We therefore multiply.*

OPERATION.

£1 6
2 0
3 2 0*s.*; since £1=20*s.*, therefore £16=320*s.*
1 2
3 8 4 0*d.*; since 1*s.*=12*d.*; 320*s.*=3840*d.*
4
1 5 3 6 0*qr.*; since 1*d.*=4*qr.*; 3840*d.*=15360*qr.*

Hence, £16=320*s.*=3840*d.*=15360*qr.* Each of these denominations are of the same value, and differ only in the number of units required of each to express that value.

2. In 15360 farthings, how many pounds?

OPERATION.

4) 1 5 3 6 0*qr.*
12) 3 8 4 0*d.*
20) 3 2 0*s.*
1 6£.

Here we bring the *value* given in the previous sum, but *expressed in a lower denomination,* back *to its original denomination by division.*

What is the difference between the two rules given? Is the given *value changed* by either operation?

162. How prove the work?

3. Reduce £15 12*s.* 8*d.* 3*qr.* to farthings.

OPERATION.

£	*s.*	*d.*	*qr.*
15	12	8	3
20			
312			
12			
3752			
4			
15011			

312=shillings in £15 12*s.*

3752=pence in 312*s.* 8*d.*

15011=farthings in 3752*d.* and 3*qr.*, and is the required answer.

We here reduce the £15 to shillings, and add the 12*s.* given; we then reduce the shillings to pence and add the given pence; and, finally, we reduce the pence to farthings, and add the given farthings. It will be observed that as we reduce the higher denominations to lower, we, in each instance, add whatever is given of the lower denomination.

4. Reduce 17685 farthings to pence.

Ans. £18 8*s.* 5*d.* 1*qr.*

5. Reduce £27 15*s.* 8*d.* to pence and farthings.

Ans. 6668*d.*, 26672*qr.*

6. In 26672*qr.* how many pounds, shillings, and pence?

Ans. £27 15*s.* 8*d.*

Observe first, with what denomination the reduction commences, and whether it is required *to bring high denominations to those of lower value, or low denominations to those of higher value.*

Observe, also, that the divisor or multiplier required in each instance is determined *by the number of units required of the given denomination to make one of the next higher or lower denomination.*

7. Reduce 17 cwt. 3 qr. 10 lb. 7 oz. to ounces.

Ans. 28567 oz.

8. Reduce 36 cwt. 2 qr. to pounds and ounces.

Ans. 3650 lb., 58400 oz.

What is to be observed? How is the divisor or multiplier to be determined?

9. In 13 lb. 17 pwt., how many grains? *Ans.* 75288 gr.
10. In 37 lb. 17 pwt., how many grains?
Ans. 213528 gr.
11. In 17897 gr., how many pounds, ounces, &c.?
Ans. 3 lb. 1 oz. 5 pwt. 17 gr.
12. Reduce 63 ℔ 9 ℥ to drams. *Ans.* 6120 ʒ.
13. In 1367 drams, how many pounds, &c.?
Ans. 14 ℔ 2 ℥ 7 ʒ.
14. Reduce 3 m. 5 fur. 8 rd. 4 yd. 2 ft. 9 in. to inches.
Ans. 231441 in.
15. Reduce 1 m. 6 fur. to inches. *Ans.* 110880 in.
16. How many square inches in a board 13 ft. 9 in. long, and 15 in. wide? *Ans.* 2475 sq. in.
17. How many square feet and inches are there in the board named in the previous sum?
Ans. 17 sq. ft. 27 sq. in.
18. How many cubic inches in 456 cu. ft. and 96 cu. in.?
Ans. 788064 cu. in.
19. In 6875988 cubic inches, how many cubic feet?
Ans. 3979 cu. ft. 276 cu. in.
20. In 17 yd. 3 qr. how many nails? *Ans.* 284 na.
21. In 21 e. E. 4 qr. and 3 na., how many nails?
Ans. 439 na.
22. In 6 hhd. 19 gal. and 2 qt. how many pints?
Ans. 3180 pt.
23. In 21 hhd. 41 gal., how many quarts?
Ans. 5456 qt.
24. In 16 bar. 31 gal. of beer, how many pints?
Ans. 4856 pt.
25. In 5 bu. 3 pk., how many pints? *Ans.* 368 pt.
26. In 736 pints, how many bushels, pecks, &c.?
Ans. 11 bu. 2 pk.
27. In 21 days, 6 hours, and 30 minutes, how many minutes? *Ans.* 30630 minutes.
28. In 30630 minutes, how many days, hours, &c.?
Ans. 21 d. 6 h. 30 m.
29. In 6 signs, 21 degrees, how many degrees and seconds?
Ans. 201 deg., 723600 sec.
30. *In 172800* seconds, how many degrees?
Ans. 48 degrees.

the forthcoming edition.—the improvement is the application of the simple rules,—in analysis and cancellation,—in explaining the principle of proportion by regarding the several terms as quotas and results, and the methods of converting square and cube roots by factors, &c., are not only ingenious, but are evidently improvements, which will be found useful in practical operations, abridging them and rendering them easy; while, at the same time, they are entirely new, plain and adapted to the youth's

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Extract of a letter from R. K. Morris, Esq., Principal of Colchester (Conn.) Academy.

In applying the principle of cancellation, as laid down in your book, the student secures even a clearer idea of his subject than by the ordinary mode of solution. I can, therefore, cheerfully express the opinion, that although several valuable treatises on Arithmetic have appeared within a few years, your work contains explanations which are rarely, if ever, to be found without resorting to different systems. The general perspicuity and conciseness of explanations and illustrations,—the completeness of the work as a system,—and the happy application of the principle of cancellation, give it strong claims on the attention of those who have the care of educating the young.

From H. S. Hanna, Esq., Principal of Academy, Troy, N. Y.

I have examined a System of Arithmetic by O. Tracy, A. M., and am satisfied that it is superior to any other work of the kind with which I am acquainted. The method of cancellation introduced must secure success to the work, being as it is well calculated to render